Using wavelets to dissect the relationships between exchange rate changes and stock returns in China

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ABSTRACT

On basis of the Wavelet analysis in time-frequency domains, the present study investigates the dynamic relationship between Chinese exchange rate changes and FTSE China A50 index returns, using day data covering from 2013-1-7 to 2020-2-3. The Wavelet analysis takes into account the examination of correlation, causality and periodicity in both time-frequency domains. The empirical results show that there are two sides causality between Exchange Rate and FTSE China A50 index. The short-run, medium and long-run cyclical effects are analyzed under a lower, intermediate as well as higher level of frequency domain. The short and medium run cyclical effects are confirmed under the levels of time-frequency domains. There are asymmetric leading or lagging relationships between first differenced form of logarithm of Exchange Rate and FTSE China A50 index.

Key words: Exchange rate changes, A50 index returns, wavelet cross spectra, wavelet coherency, wavelet phase difference.

INTRODUCTION

The stock market is a barometer of the economy, where the rapid flows of capital leads to stock prices changes. The exchange rate market is also affected by the capital flows which results in exchange rate fluctuation. It means that there might be a relationship between stock returns and exchange rate changes.

The relationship between stock returns and exchange rate changes has been studied widely and has been applied to variables causality domestic and abroad. According to previous literatures, this causality maybe subdivided two branches. The first is that there is a one-way causality between stock returns and exchange rate changes (Branson, 1983; Dornbusch and Fisher, 1980). The second is that there are two-way causalities between them (Granger et al., 2000). As a result, some interesting causalities exist between stock returns and exchange rate changes at different time scales. According to the results on time series model, Kanas (1997) investigates the interdependence of stock returns and exchange rate changes and finds evidence of spillovers from stock returns to exchange rate changes for most countries. Doukas et al. (1999) find a significant relationship between stock returns and yen fluctuations for firms traded on the Tokyo Stock Exchange. Dewenter et al. (2005) examine event studies and find evidence in support of a contemporaneous relationship between stock returns and exchange rates. Many domestic scholars find the long cointegration relationship between stock returns and exchange rate changes on China, using VAR models.

Furthermore, the asymmetric effects of exchange rate changes on stock returns have been another interesting area of research. Exchange rate appreciation may have a different impact on stock returns as compared with the

As compared with the traditional econometric methods, a Wavelet Coherency method is a more advanced econometric method. It can not only effectively capture the structural changes in time domains, but also capture the short, medium and long run coherency in frequency domains over time, and reveal the leading or lagging relationship between variables. Generally, the causal and reverse causal relationship between variables can be estimated using the Wavelet analysis. Aguiar et al. (2008) use wavelet tool to show that the relationship between interest rates and M1 has changed and evolved with time. Gallegati (2008) investigates the relationship between stock market returns and economic activity by employing signal decomposition techniques based on wavelet analysis. He shows that stock market returns tend to lead the level of economic activity, but only at the highest scales (lowest frequencies) corresponding to periods of 16 months and longer, and that the leading period increases as the wavelet time scale increases. Tiwari (2014) finds leading or lagging relationship between interest rate and share prices. Su et al. (2016) find the positive correlation between stock prices and exchange rate in China, revealing that stock prices lead to the fluctuation in exchange rate in time domain, and correlate with exchange rate in short run in frequency domain.

However, a few of studies on the relationship between stock returns and exchange rate changes have been discussed using wavelet tools. In the present study, the wavelet tools are applied to the time-frequency relationships between stock returns and exchange rate changes. The leading or lagging relations are built and the intervals periods on lead or lag are also estimated. We focus on the short and medium run cyclical effects between variables.

The study proceeds as follows: the wavelet tools are introduced to construct the theory basis, followed by data and analysis of the results, and then conclusion.

**METHODS**

Wavelet analysis performs the estimation of the spectral characteristics of a time series as a function of time, showing how the different periodic components of the time series change over time. One major advantage accorded by the wavelet transform is the ability to perform natural local analysis of a time series: the wavelet stretches into a long function to measure the low frequency movements, and it compresses into a short function to measure the high frequency movements.

### The continuous wavelet transform (CWT)

A wavelet is a function with zero mean and that is localized in both frequency and time. We can characterize a wavelet by how localized it is in time \((\Delta t)\) and frequency \((\Delta \omega)\) or the bandwidth). The classical version of the Heisenberg uncertainty principle tells us that there is always a tradeoff between localization in time and frequency. Without properly defining \(\Delta t\) and \(\Delta \omega\), we will observe that there is a limit to how small the uncertainty product \(\Delta t \cdot \Delta \omega\) can be. One particular wavelet, the Morlet, is always defined as:

\[
\varphi_\omega(\eta) = \pi^{-1/4} e^{i \omega_0 \eta} e^{-\frac{1}{2} \eta^2}
\]  

(1)

where \(\omega_0\) is the dimensionless frequency and \(\eta\) is the dimensionless time. When using wavelets for feature extraction purposes, the Morlet wavelet (with \(\omega_0=6\)) is a good choice, since it provides a good balance between time and frequency localization. We therefore restrict our further treatment to this wavelet, although the methods we present are generally applicable (Foufoula-Georgiou, 1995).

### Wavelet tools

#### Wavelet power spectra

We simply define the wavelet power spectrum as \(|W_R|^2\), which gives us a measure of the local variance. The statistical significance of wavelet power can be assessed against the null hypothesis that the data generating process is given by a stationary process with a certain background power spectrum \((P_f)\). Torrence and Compo (1998) compute the white noise and red noise wavelet power spectra, from which they derived, under the null, the corresponding distribution for the local wavelet power spectrum,

\[
\frac{|W_R|^2}{\sigma^2} < p \frac{1}{2} \frac{1}{v} \chi_v^2
\]  

(2)
at each time \( n \) and scale \( s \). The value of \( P_f \) is the mean spectrum at the Fourier frequency \( f \) that corresponds to the wavelet scale \( s \) – in our case \( s \approx 1/f \), and \( v \) is equal to 1 or 2, for real or complex wavelets respectively. For more general processes, one has to rely on Monte Carlo simulations.

**Cross-wavelet power**

The cross-wavelet transform of two time series, \( x = \{X_n\} \) and \( y = \{Y_n\} \), first introduced by Hudgins et al., is simply defined as:

\[
W^{xy}_n = W^x_n W^{y*}_n
\]  

(3)

where \( W^x_n \) and \( W^{y*}_n \) are the wavelet transforms of \( x \) and \( y \), respectively. The cross-wavelet power is given by \( |W^{xy}_n| \). While we can interpret the wavelet power spectrum as depicting the local variance of a time series, and the cross-wavelet power of two time series depicts the local covariance between these time series at each scale or frequency. Therefore, cross-wavelet power gives a quantified indication of the similarity of power between two time series. For white and red noise processes, Torrence and Compo (1998) show that if two time series have Fourier Spectra \( P^x_f \) and \( P^y_f \), then the cross-wavelet distribution is given by:

\[
\frac{|W^x_n W^{y*}_n|}{\sigma_x \sigma_y} \leq \frac{z_v(p)}{\sqrt{P^x_f P^y_f}}
\]  

(4)

where \( z_v(p) \) is the confidence level associated with the probability \( p \) for a pdf defined by the square root of the product of two \( \chi^2 \) distributions. For more general data generating processes, one has to rely on Monte Carlo simulations.

**Wavelet coherency**

In the Fourier spectral approaches, wavelet coherency can be defined as the ratio of the cross-spectrum to the product of the spectrum of each series, and can be thought of as the local correlation, both in time and frequency, between two time series. We define the wavelet coherency between two time series \( x = \{X_n\} \) and \( y = \{Y_n\} \) as follows:

\[
R_n(S) = \left| \frac{\mathcal{S}(s^{-1} W^x_n W^{y*}_n)(s))}{\mathcal{S}(s^{-1} W^x_n(s))^{1/2} \mathcal{S}(s^{-1} W^{y*}_n(s))^{1/2}} \right|
\]  

(5)

where \( S \) denotes a smoothing operator in both time and scale. Smoothing is necessary. Without that step, coherency is identically 1 at all scales and times. Smoothing is achieved by a convolution in time and scale. The time convolution is done with a Gaussian and the scale convolution is performed with a rectangular window. Theoretical distributions for wavelet coherency have not been derived yet. Therefore, to assess the statistical significance of the estimated wavelet coherency, one has to rely on Monte Carlo simulation methods.

**Phase difference**

As Soares et al. (2011) explain, this tool provides information about the delays of the oscillations between two time series, \( x = \{X_n\} \) and \( y = \{Y_n\} \), as a function of frequency. As mentioned earlier, the phase of a given time series \( \varphi_x \) can be viewed as the position in the pseudo-cycle of the series. The phase difference \( \varphi_{xy} \) characterizes phase relationships between the two-time series, that is, their relative position in the pseudo-cycle. The phase difference is defined as:

\[
\varphi_{xy} = \tan^{-1} \left( \frac{\{W^x_n\}}{R\{W^y_n\}} \right) \quad \text{with} \quad \varphi_{xy} \in [-\pi, \pi].
\]  

(6)

A phase difference of zero indicates that the time series move together at specified frequency. If \( \varphi_{xy} \in (0, \pi/2) \) then the series move in phase, but the time series \( x \) leads \( y \). If \( \varphi_{xy} \in (-\pi/2, 0) \) then it is \( y \) that is leading. A phase difference of \( \pi \) (or \( -\pi \)) indicates an anti-phase relation. If \( \varphi_{xy} \in (\pi/2, \pi) \) then \( y \) is leading. Time series \( x \) is leading if \( \varphi_{xy} \in (-\pi, -\pi/2) \).

**Cross wavelet phase angle**

As we are interested in the phase difference between the components of the two-time series, we need to estimate the mean and confidence interval of the phase difference. We use the circular mean of the phase over regions with higher than 5% statistical significance that are outside the COI to quantify the phase relationship. This is a useful and general method for calculating the mean phase. The circular mean of a set of angles \( \{\alpha_i, i=1..n\} \) is defined as (e.g. Zar, 1999):
It is difficult to calculate the confidence interval of the mean angle reliably since the phase angles are not independent. The number of angles used in the calculation can be set arbitrarily high simply by increasing the scale resolution. However, it is interesting to know the scatter of angles around the mean. For this, we define the circular standard deviation as:

$$a_m = \arg(X, Y) \quad \text{with} \quad X = \sum_{i=1}^{n} \cos(a_i), \quad Y = \sum_{i=1}^{n} \sin(a_i) \quad (7)$$

where $R = \sqrt{X^2 + Y^2}$.

The circular standard deviation is analogous to the linear standard deviation in that it varies from zero to infinity. It gives similar results to the linear standard deviation when the angles are distributed closely around the mean angle. In some cases, there might be reasons for calculating the mean phase angle for each scale, and then the phase angle can be quantified as a number of days.

**STOCK RETURNS AND EXCHANGE RATE CHANGES**

For each of analysis proceeding, we obtain data of A50 stock index (sti) and exchange rate (exe) with daily observations covering from 2013-1-7 to 2020-2-3, all data are available at the Wind Database. Ajayi et al (1998) first use the natural logarithm of data shown in Figure 1, and transform the variables in first difference form shown in Figure 2.
first differenced stock index can be viewed as stock returns, the first differenced exchange rate stands for the changes. According to the results in Table 1, the stationary series can be testified.

The CWT power spectra of the exchange rate changes and A50 index returns for daily data are shown in Figure 3. From Figure 3a and 1b, we can observe the common power features in short or medium run corresponding to 2013-8 to 2017-12. Both series have higher power fluctuations during this period of scale, and for both series the power is significant at the 5% level, respectively. Though for both series in short and medium run, the power is above the 5% significance level, the common features have significant differences on the distribution. The cross-wavelet transform maybe help in this regard.

To obtain the confirmed causality relationships between

Table 1: ADF tests.

<table>
<thead>
<tr>
<th></th>
<th>P-Value</th>
<th>Statistics</th>
<th></th>
<th>P-Value</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnexe_t</td>
<td>0.8176</td>
<td>0.7864</td>
<td>Δln exe_t</td>
<td>0.0001</td>
<td>-47.45</td>
</tr>
<tr>
<td>lnsti_t</td>
<td>0.8834</td>
<td>0.5521</td>
<td>Δln sti_t</td>
<td>0.0001</td>
<td>-47.63</td>
</tr>
</tbody>
</table>
the two variables, we expect that the oscillations are phase-locked. The XWT shows that the phase between ln(sti) returns and ln(exe) changes with significant common features, as shown in Figure 4. The two variables are out of phase in the 70-128 days of scale from 2013-12 to 2015-6 and arrows are left and up. This indicates that ln(sti) returns are leading and the mean phase angle is 0.4839, the average time intervals are about 27 days. They are out of

Figure 3: Continue wavelet power spectra.
(a). Wavelet power spectrum—The white contour designates the 5% significance level against a white noise null. The black contour designates the 5% significance level estimated from Monte Carlo simulations using phase randomized surrogate series. The cone of influence, which indicates the region affected by edge effects, is shown with a solid line. The color code for power ranges from blue (low power) to yellow (high power).
(b). Wavelet power spectrum—The white contour designates the 5% significance level against a white noise null. The black contour designates the 5% significance level estimated from Monte Carlo simulations using phase randomized surrogate series. The cone of influence, which indicates the region affected by edge effects, is shown with a solid line. The color code for power ranges from blue (low power) to yellow (high power).

a. Ln(sti) returns

b. Ln(exe) changes
Figure 4: XWT-- ln(exe) changes and ln(sti) returns.
Cross wavelet transforms between the first differenced ln(exe) and ln(sti) time series. The 5% significance level against red noise is shown as a thick contour. The relative phase relationship is shown as arrows (with in-phase pointing right, anti-phase pointing left). The color code for power ranges from blue (low power) to yellow (high power). Arrows pointing to the right and up, with first differenced ln(sti) lagging. To the right and down, with first differenced ln(sti) leading. Arrows pointing to the left and up, with first differenced ln(sti) leading. To the left and down, with first differenced ln(sti) lagging.

Figure 5: WTC--ln(exe) changes and ln(sti) returns.
Squared wavelet coherency between the first differenced ln(exe) and ln(sti) time series. The 5% significance level against red noise is shown as a thick contour. The relative phase relationship is shown as arrows (with in-phase pointing right, anti-phase pointing left). The color code for coherency ranges from blue (low coherency—close to zero) to yellow (high coherency—close to one). Arrows pointing to the right and up, with first differenced ln(sti) lagging. To the right and down, with first differenced ln(sti) leading. Arrows pointing to the left and up, with first differenced ln(sti) leading. To the left and down, with first differenced ln(sti) lagging.

phase in the 16-28 days of scale from 2016-8 to 2016-11 and arrows are right and up. This indicates that ln(sti) returns are lagging and the mean phase angle is -0.8118, the average time intervals are about 46 days. At the similar intervals, the leading days are not consistent with each other, this indicates that it is necessary to consider the wavelet coherency and phase difference relationships between them.

The wavelet coherency and mean phase difference of between ln(sti) returns and ln(exe) changes with 5% significance level as shown in Figure 5. As compared with the XWT, many short run and medium run relations are significant, and these areas present the complicated causalities between them. Nine significant time-frequency
contours are plotted and the reliable indication of causalities are shown. The correlation coefficients of two series are above 0.9 in the nine significant contours in the time-frequency spaces.

In short run frequency band, from 2013-6 to 2013-8, the arrow pointing to left and up indicates the ln(exe) changes lead ln(sti) returns, the mean phase difference of 0.9654 in 19-28 days of scale means ln(sti) changes have been ahead of ln(exe) returns for 55.1 days. It means that the capital flow out of stock market and inflow into exchange market. The stock index finally decreased and exchange rate kept strong in this period. From 2015-6 to 2015-10, the arrow pointing to left and up indicates the ln(exe) changes lag ln(sti) returns, the mean phase difference of 0.4416 in 16-25 days of scale means ln(sti) changes have been ahead of ln(exe) returns for 25.2 days. This short run frequency band means that the capital flow out of stock market and flow into exchange market. The stock market dropped down to 2015-8 since June, exchange rate kept strong, but from August to October stock market began to rise, and exchange rate depreciated.

From 2015-11 to 2016-2, the arrow pointing to left and down indicates the ln(exe) changes lag ln(sti) returns, the mean phase difference of 0.142 in 14-19 days of scale means ln(exe) changes have been ahead of ln(sti) returns for 8.14 days. This short run frequency band also means that the capital flow out of exchange market and flow into stock market. This pushed stock market to a new peak on 2015-12-12, exchange rate began to depreciate. From 2016-6 to 2016-7, the arrow pointing to right and down indicates the ln(exe) changes lag ln(sti) returns, the mean phase difference of 0.8798 in 19-27 days of scale means ln(sti) changes have been ahead of ln(exe) returns for 50.2 days. It means that the capital flow out of stock market and inflow into exchange market in short-run scales. This contributed to stock market falls and pushed an increase in exchange rate.

From 2016-12 to 2017-2, the arrow pointing to left and down indicates the ln(exe) changes lead ln(sti) returns, the mean phase difference of 0.747 in 20-28 days of scale means ln(exe) changes have been ahead of ln(sti) returns for 42.8 days. This means that the capital flow out of exchange market and inflow into stock market. From 2017-12 to 2018-2, the arrow pointing to left and down indicates the ln(exe) changes lead ln(sti) returns, the mean phase difference of 0.9466 in 20-28 days of scale means ln(exe) changes have been ahead of ln(sti) returns for 55.1 days. This means that the capital flow out of exchange market and inflow into stock market in short-run scales. The stock market eventually went up and exchange rate got weak in these periods.

From 2018-11 to 2019-1, the arrow pointing to left and up indicates the ln(exe) changes lag ln(sti) returns, the mean phase difference of 0.449 in 20-30 days of scale means ln(exe) changes lag behind ln(sti) returns for 25.7 days. It means that the capital flow out of stock market and inflow into exchange market in short-run scales. This also led to falls on stock market and exchange rate appreciation.

In terms of medium-run scales, from 2017-3 to 2017-9, the arrow pointing to left and down indicates the ln(exe) changes lead ln(sti) returns, the mean phase difference of 0.36 in 31-45 days of scale means ln(exe) changes have been ahead of ln(sti) returns for 20.6 days. It means that the capital flow out of exchange market and inflow into stock market, which is not consistent with real stock index and real exchange rate fluctuation. The next period should be considered. From 2019-5 to 2019-11, the arrow pointing to left and down indicates the ln(exe) changes lead ln(sti) returns, the mean phase difference of 0.572 in 48-70 days of scale means ln(exe) changes have been ahead of ln(sti) returns for 32.7 days. It means that the capital flow out of exchange market and inflow into stock market in medium-run scales. This also led to rise on stock market and exchange rate depreciation.

From the results of short and medium run time-frequency domains, we find a capital flow curve, such as stock market—exchange market—stock market—exchange market—stock market-exchange market. Capital inflows bring the expected returns and push the stock market rise firstly, then exchange rate seems to depreciate by an outflow of capital, and vice versa.

Conclusion

In this study, the asymmetric causalities between A50 Index and Exchange rate are confirmed in short and medium run scales in time-frequency domains. They show a new trend on capital flows from stock market to exchange market or from exchange market to stock market.

According to seven significant levels of time-frequency domains in short run scales, cycle periods of four of seven are twice as much as the short run scales of themselves, respectively. This means that capital flows lack efficiency. Stock falling and exchange rate rising may be caused by the problem on regulation of stock market, a Stock not entering into MSCI and bilateral currency swap agreements with the West in first two scales. Stock rising and exchange rate falling may be caused by floating exchange rate and more intrusive surveillance on stock market in latter two scales. The remaining three periods are more efficient in short run scales, their cycle periods are within the short run scales. Exchange rate reform of 2015 led to the depreciation of exchange rate, but stock market was pushed to peak by leverage of securities margin trading. Although an excessive leverage ratio caused a crash on stock market in the second half of 2015, the stock market climbed again as government implemented a rescue plan in Oct. 2015, the exchange rate kept declining. The exchange rate kept rising because of contra-cyclical factors rebooting after November, 2018, the stock market fell down because of trade friction on China-USA and delisting of Chang Sheng.
In term of medium run scales, stock market kept rising as the settlement of trade friction between China and USA becomes a big probability event. The exchange rate kept declining with strong real growth of USA economy.

Regarding the complicated economic variables, more variables may be placed into this model, such as using the real estate market to struck the partial wavelet coherency or multiple wavelet coherency. This led to the discovery of some new breakthrough points for future researches.

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REFERENCES


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