A study on optimal ordering policies under different lead time and heterogeneous distribution

Accepted 15th June, 2020

ABSTRACT

This study assumes that the lead time is less than seven days, so the length of lead time and the different stochastic demand distributions will have an influence on the reorder point. Based on the electronic transaction system (ETS) to collect the sales data, then we set up an inventory cost model under given lead time and to get the optimal ordering policies by applying optimization techniques to minimize the total expected inventory cost. Therefore, through the use of cluster analysis method and goodness of fit test to predict the demand distribution of each group and then construct the heterogeneous demand distribution of lead time. A numerical example was given to verify the correctness of the proposed inventory cost model, and the sensitivity analysis was taken to realize the influence of related important parameters. Finally, conclusions are drawn for practical applications and future studies.

Key words: Stochastic demand, lead time, reorder point.

INTRODUCTION

The inventory is the main factor on how to gain competitive advantages in the market for retailers, so, good inventory management is an important decision for any company. In traditional economic order quantity (EOQ) model supposed that during the planning period is known, and the demand is viewed as a constant. However, in the stores, the demand of merchandise will be influenced by the length of lead time and lead time across the number of daily sales.

Generally speaking, the demand distribution of lead time was assumed as a distribution, so the daily reorder point is fixed. However, this study will be divided based on a week into non-holiday and holiday, and the different demand distributions of non-holiday and holiday to get the reorder points, respectively. Therefore, the merchandise sales data will be clustered through by goodness of fit test, and then we can obtain the demand distribution of non-holiday and holiday, respectively. This research explored whether the distribution with additive property, and set up an inventory decision model. In addition, a numerical analysis method is developed to obtain the optimal ordering policies to minimize the total expected inventory cost.

LITERATURE REVIEW

We collected the transaction data that will be clustered and through by goodness of fit test, to explore whether each distribution has additive. Generally speaking, inventory model for economic order quantity or economic production quantity has been studied for years. They added some constraints to models such as, the stochastic demand, price discount and so on.

Cluster analysis

Cluster analysis represents a wide range of techniques used for combining cases into groups or clusters with similar characteristics. It can be applied in various areas of research such as computer science, bio-information, marketing, manufacturing, psychology, and so on. In terms of the classification method, it can be divided into hierarchical cluster analysis and non-hierarchical cluster analysis method.
Tou and Gonzalez (1974) proposed the K-Means cluster analysis method which is widely used for cluster method. Yucel and Demir (2004) studied cluster method and reported it to be a useful approach for the characterization of marbles based on chemical properties. Székely and Rizzo(2005) have recently described a new hierarchical agglomerative clustering method. This method generalizes Ward’s method by defining a cluster distance and objective function in terms of a power in the interval (0,2] of the Euclidean distance between cluster centers, with Ward’s method being obtained as the limiting case when the power is 2. Boyacioglu and Boyacioglu (2008) studied the cluster method which was applied to assess water quality.

Goodness of fit test

Goodness of fit test is a test method, which uses sample data to judge whether population distribution is a theoretic distribution or not. There are a lot of goodness of fit tests such as the chi-square test, the Kolmogorov-Smirnov test, the Anderson-Darling test and the Cramér-Von Mises test and so on.

Dahiya and Gurland (1972) used the minimum chi-square technique for groups of data to construct a test. Harter (1980) applied the interpolation method to obtain a critical value of K-S test, it generally provided for K-S test by the text book. Thas and Ottoy (2003) pointed out the Anderson-Darling statistic is basically a weighted average of Pearsonchi-square statistics. Liang (2004) proposed the Cramér-Von Mises test to check the difference between two regression curves, their test is easily understood, and does not impose restrictive assumption on the regression curves. Roberts (2019) proved that after the rotation to the non-distribution process, the goodness of fit test is easy to calculate and has a high power to reject incorrect null hypotheses.

Stochastic demand

The traditional EOQ model assumed that the demand is fixed, in fact, the demand is a random variable, and many researchers have added the condition of stochastic demand to inventory models. For example, Hadley and Whitin (1963) proposed an inventory model for a single product with stochastic demand, and shortage is allowed. Browne and Zipkin (1990) considered the (r,Q)inventory model where demand accumulates continuously, but the demand rate at each instant was found by an underlying stochastic process. Alfares (2007) proposed the inventory model with stock-level dependent demand rate and variable holding cost. Hsieh et al. (2010) proposed a deterministic inventory model with which time-dependent backlogging rate is developed. The demand rate is a power function of the on-hand inventory down to a certain stock level, at which the demand rate becomes a constant.

Inventory model

The Economic Order Quantity (EOQ) model initiated by Harris has been extended in many ways to improve the real model. However, the model assumption cannot solve in the actual situation, such as some conditions for the demand with non-fixed constants, this made researchers develop EOQ models in many conditions. In terms of inflation, Misra (1979) proposed an inflation model for the EOQ, in which the time value of money and different inflation rates were considered. Chang (2004) established an EOQ model for deteriorating items under inflation when the supplier offers a permissable delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. Chern et al., (2008) developed the inventory lot size models for deteriorating items with fluctuating demand under inflation. Mokhtari (2018) suggested that in the actual situation, if the product is out of stock, some products complement each other. Therefore, the Economic Order Quantity (EOQ) model was discussed to determine the joint ordering policy for both products under completion and replacement conditions to optimize the total cost of inventory.

Reorder point

Zipkin (2000) stated that there are two major decisions in inventory management at a single location, how much to order(order quantity) and when to order (reorder point) it. The reorder point is defined as a series of process when the existing stock cannot meet before the ordering arrival. In general, the reorder point is the key on the demand and lead time. Chang and Yao, (1998) solved the economic reorder point with fuzzy backorders. There were some articles addressing fuzzy demands. Axsäter (2003) proposed a two-echelon distribution inventory system with a central warehouse and a number of retailers. The retailers face stochastic demand, the system is controlled by continuous review installation stock (R, Q) policies with given batch quantities, he also presented a simple technique for approximate optimization of the reorder points. Chaharsosghii and Heydari (2010) pointed out that the aim of the proposed coordination model is to coordinate the reorder point in addition to the order quantities in a two-level SC with a backorder inventory model, and to show that the coordination of the reorder point with order quantity can increase chain profitability. Sevgen and Sargut, (2019) Created a mathematical model to determine the best parameters for the retailer's most-type of order strategy, and also studied the importance of the retailer’s non-zero reorder point, determined when non-zero reorder points are cost effective, and combine
solutions with classics EOQ is compared.

MODEL CONSTRUCTION

This chapter is divided into four sections. First, whether a given distribution has additive property or not. Second, the optimal total inventory model of general merchandise is constructed, and then a numerical analysis method is proposed to calculate the optimal merchandise ordering policy in order to minimize the total inventory cost. Finally, sensitivity analysis for related parameters is taken to understand the influence on total inventory cost and decision variables.

Distribution with additive

In general, merchandise demand is not necessarily followed by a normal distribution, only by collecting sales data to obtain the mean and variance of demand. We assume that it can follow any distribution of daily demand when the lead time is greater than one day, the daily demand must be cumulated and viewed as the demand of lead time, involving random variables with additive. The follows are some common distributions shown in Table 1.

Assumptions

This article has the following assumptions:

1. Based on electronic transaction system to obtain the merchandise sales data.
2. It only considered the general merchandise, which means the merchandise is not perishable goods.
3. The ordering cost, the holding cost and the shortage cost are known.
4. The lead time is known and fixed, given range from 1 to 6 days.
5. The planning period is limited and known.
6. Without considering quantity discounts and inflation.
7. Suppose a week can be divided into non-holiday and holiday.
8. Demand is a random variable.

Inventory cost model construction

To get the optimal order quantity and reorder point in order to minimize the total expected inventory cost. Generally speaking, the total expected inventory cost of general merchandise, including total ordering cost, total holding cost and total shortage cost. Therefore, in terms of the complete planning cycle, the total expected inventory cost of general merchandise is expressed as follows:

The total inventory cost = the total ordering cost + the total holding cost + the total shortage cost

\[
E(C_{LT}) = \frac{E(Y_{LT})}{N_{LT}} \cdot O_c + \left[ \frac{N_{LT}}{2} \cdot H_c + \int_{\theta_{LTq} - ROP_{LTq}}^{\theta_{LTq}} (ROP_{LTq} - \theta_{LTq}) \cdot H_c \cdot f(\theta_{LTq}) \cdot d\theta_{LTq} \right] + \frac{E(Y_{LT})}{N_{LT}} \cdot \int_{ROP_{LTq}}^{\infty} \left( \theta_{LTq} - ROP_{LTq} \right) \cdot S_c \cdot f(\theta_{LTq}) \cdot d\theta_{LTq}
\]

Because of this, sales on Friday are different by product and industry, while Friday belongs to non-holiday or holiday, rigorous statistical methods must be applied to analyze and understand.
If the clustering result shows that Friday belongs to non-holiday, the demand distribution under the different lead time, where the reorder point has two or three different quantities. When \( LT = 1 \), it has two reorder points, including one day of non-holiday and one day of holiday, where \( \sum_{q=0}^{1} M_{iq} = M_{10} + M_{11} = \frac{5}{7} + \frac{2}{7} = 1 \). When \( LT = 2 \), it has three reorder points, including two days of non-holiday and two days of holiday. When \( LT = 3 \), it has four reorder points, including three days of non-holiday and one day of holiday. When \( LT = 5 \), it has five reorder points, including four days of non-holiday and one day of holiday. When \( LT = 6 \), it has six reorder points, including two days of holiday and three days of holiday. Hence, we take \( LT = 1 \) as an example.

If \( LT = 1 \), the demand distribution can be divided into one day of non-holiday and one day of holiday, namely

**One day of non-holiday:**

\[
EC_{ii} = \frac{E(Y_{i})}{N_{i}} \cdot O_{i} + M_{i1} \left[ \frac{N_{i}}{2} \cdot H_{i} + \int_{\theta_{i1}}^{\infty} (ROP_{i1} - \theta_{i1}) \cdot H_{i} \cdot f(\theta_{i1}) \cdot d\theta_{i1} \right] + \frac{E(Y_{10})}{N_{1}} \cdot \int_{\theta_{10} = ROP_{10}}^{\infty} (\theta_{10} - ROP_{10}) \cdot S_{c} \cdot f(\theta_{10}) \cdot d\theta_{10}
\]

**One day of holiday:**

\[
EC_{ii} = \frac{E(Y_{i})}{N_{i}} \cdot O_{i} + M_{i1} \left[ \frac{N_{i}}{2} \cdot H_{i} + \int_{\theta_{i1}}^{\infty} (ROP_{i1} - \theta_{i1}) \cdot H_{i} \cdot f(\theta_{i1}) \cdot d\theta_{i1} \right] + \frac{E(Y_{11})}{N_{1}} \cdot \int_{\theta_{11} = ROP_{11}}^{\infty} (\theta_{11} - ROP_{11}) \cdot S_{c} \cdot f(\theta_{11}) \cdot d\theta_{11}
\]

Hence, the total expected inventory cost is:

\[
EC = \left( \frac{E(Y_{i})}{N_{i}} \cdot O_{i} + M_{i1} \left[ \frac{N_{i}}{2} \cdot H_{i} + \int_{\theta_{i1}}^{\infty} (ROP_{i1} - \theta_{i1}) \cdot H_{i} \cdot f(\theta_{i1}) \cdot d\theta_{i1} \right] \right) + \frac{E(Y_{10})}{N_{1}} \cdot \int_{\theta_{10} = ROP_{10}}^{\infty} (\theta_{10} - ROP_{10}) \cdot S_{c} \cdot f(\theta_{10}) \cdot d\theta_{10}
\]

In order to minimize the total expected inventory cost \( EC_{i} \), taking the first partial derivatives of \( EC_{i} \) with respect to \( N_{1}, ROP_{10} \) and \( ROP_{11} \), where applying the Leibniz's integral rule (see Appendix A for the proof):

\[
+ \frac{E(Y_{i1})}{N_{i}} \cdot \frac{N_{i}}{2} \cdot H_{i} + \int_{\theta_{i1}}^{\infty} (ROP_{i1} - \theta_{i1}) \cdot H_{i} \cdot f(\theta_{i1}) \cdot d\theta_{i1}
\]

Setting Equations (5), (6) and (7) to zero and solving for \( N_{1} \), \( ROP_{10} \) and \( ROP_{11} \), it follows that:

\[
N^{*}_{1} = \left\{ \left[ \frac{2 \cdot O_{i} \cdot E(Y_{i}) + \sum_{q=0}^{1} (E(Y_{iq}) \cdot S_{c} \cdot E(Y_{iq}))}{C_{k}} \right] + \frac{1}{2} \right\}^{\frac{1}{2}}
\]

\[
F(ROP_{10}) = \left[ \frac{S_{c} \cdot E(Y_{10})}{M_{10} \cdot H_{i} \cdot N_{1} + S_{c} \cdot E(Y_{10})} \right]
\]

and

\[
F(ROP_{11}) = \left[ \frac{S_{c} \cdot E(Y_{11})}{M_{11} \cdot H_{i} \cdot N_{1} + S_{c} \cdot E(Y_{11})} \right]
\]

We applied numerical analysis methods to obtain the
optimal values of $N_t$, $ROP_{10}$ and $ROP_{11}$, because of Equation(8), (9) and (10) are all non-linear functions, and not easy to use algebraic methods to solve.

Taking the second order partial derivatives:

$$
\frac{\partial^2 EC_1}{\partial N_t^2} = \frac{1}{N_t^2} \{ a \cdot E(Y_t) + \sum_{t=0}^{\infty} E(Y_{t+1}) \cdot \sum_{\alpha=0}^{\infty} (\theta_{t+\alpha} - ROP_{t+\alpha}) \cdot f(\theta_{t+\alpha}) \cdot d\theta_{t+\alpha} \} > 0
$$

$$
\frac{\partial^2 EC_1}{\partial ROP_{10}^2} = \frac{\partial}{\partial ROP_{10}} \left[ M_{10} \cdot H_c + \frac{E(Y_{t+1}) \cdot S_c}{N_t} \right] > 0
$$

$$
\frac{\partial^2 EC_1}{\partial ROP_{11}^2} = \frac{\partial}{\partial ROP_{11}} \left[ M_{11} \cdot H_c + \frac{E(Y_{t+1}) \cdot S_c}{N_t} \right] > 0
$$

$$
\frac{\partial^2 EC_1}{\partial N_t \partial ROP_{10}} = \frac{E(Y_{t+1}) \cdot S_c}{N_t^2} [1 - F(ROP_{10})] > 0
$$

$$
\frac{\partial^2 EC_1}{\partial N_t \partial ROP_{11}} = \frac{E(Y_{t+1}) \cdot S_c}{N_t^2} [1 - F(ROP_{11})] > 0
$$

and

$$
\frac{\partial^2 EC_1}{\partial ROP_{10} \partial ROP_{11}} = 0
$$

It can be easily verified that $EC_1$ is a convex function of $(N_t, ROP_{10}, ROP_{11})$ and minimized the total expected inventory cost.

In addition, the Hessian matrix of $EC_1$ for a given value of $LT = 1$ can be shown as:

$$
H(N_t, ROP_{10}, ROP_{11}) = \begin{bmatrix}
\frac{\partial^2 EC_1}{\partial N_t^2} & \frac{\partial^2 EC_1}{\partial N_t \partial ROP_{10}} & \frac{\partial^2 EC_1}{\partial N_t \partial ROP_{11}} \\
\frac{\partial^2 EC_1}{\partial N_t \partial ROP_{10}} & \frac{\partial^2 EC_1}{\partial ROP_{10}^2} & \frac{\partial^2 EC_1}{\partial ROP_{10} \partial ROP_{11}} \\
\frac{\partial^2 EC_1}{\partial N_t \partial ROP_{11}} & \frac{\partial^2 EC_1}{\partial ROP_{11} \partial ROP_{10}} & \frac{\partial^2 EC_1}{\partial ROP_{11}^2}
\end{bmatrix}
$$

where

$$
\frac{\partial^2 EC_1}{\partial N_t^2} > 0
$$

$$
\frac{\partial^2 EC_1}{\partial ROP_{10}^2} = f(ROP_{10}) \left[ M_{10} \cdot H_c + \frac{E(Y_{t+1}) \cdot S_c}{N_t} \right] > 0
$$

$$
\frac{\partial^2 EC_1}{\partial ROP_{11}^2} = f(ROP_{11}) \left[ M_{11} \cdot H_c + \frac{E(Y_{t+1}) \cdot S_c}{N_t} \right] > 0
$$

$$
\frac{\partial^2 EC_1}{\partial N_t \partial ROP_{10}} = \frac{E(Y_{t+1}) \cdot S_c}{N_t^2} [1 - F(ROP_{10})] > 0
$$

$$
\frac{\partial^2 EC_1}{\partial N_t \partial ROP_{11}} = \frac{E(Y_{t+1}) \cdot S_c}{N_t^2} [1 - F(ROP_{11})] > 0
$$

and

$$
\frac{\partial^2 EC_1}{\partial ROP_{10} \partial ROP_{11}} = 0
$$

Next, we proceed by evaluating the determinant of $H$ at point $(N_t, ROP_{10}, ROP_{11})$. The first order determinant of $H$ is:

$$|H_1| = \frac{\partial^2 EC_1}{\partial N_t^2} > 0 \quad (15)$$

The second order determinant of $H$ is:

$$|H_2| = \left[ \frac{\partial^2 EC_1}{\partial N_t^2} \right] \cdot \left[ \frac{\partial^2 EC_1}{\partial ROP_{10}^2} \right] - \left[ \frac{\partial^2 EC_1}{\partial N_t \partial ROP_{10}} \right]^2 > 0 \quad (16)$$

The third order determinant of $H$ is:

$$|H_3| = \left[ \frac{\partial^2 EC_1}{\partial N_t^2} \right] \cdot \left[ \frac{\partial^2 EC_1}{\partial ROP_{10}^2} \right] \cdot \left[ \frac{\partial^2 EC_1}{\partial ROP_{11}^2} \right] + 2 \cdot \left[ \frac{\partial^2 EC_1}{\partial N_t \partial ROP_{10}} \right] \cdot \left[ \frac{\partial^2 EC_1}{\partial ROP_{10} \partial ROP_{11}} \right] \cdot \left[ \frac{\partial^2 EC_1}{\partial N_t \partial ROP_{11}} \right]$$
Therefore, from Equations (18), (19) and (20), it is clearly seen that the Hessian matrix \( \mathbf{H} \) is positive definite at point \( (N^*_1, ROP_{10}^*, ROP_{11}^*) \).

All of the above are directed to \( LT = 1 \) that explores the optimal ordering strategy, and the different situations are derived as same way as \( LT = 1 \). The follows are some optimal decision variables and reorder points as shown in Table2 to Table9:

**Sensitivity analysis**

This part of the study brings the known information into the model and conducts sensitivity analysis to realize the influence of parameters, so we are taking \( LT = 1 \) for example.

**The \( N^*_1 \) and \( ROP_{1q}^* \) influenced by the changes of \( S_c, H_c \) and \( O_c \)**

1. When \( S_c \) increases:

   The result has two situations. For one thing, when \( S_c \) increases, we can see that the order quality \( N^*_1 \) will increase from Equation (8), the order quality \( N^*_1 \) increases that will make the reorder point \( ROP_{1q}^* \) decrease from Equation (9) and (10), but the reorder point \( ROP_{1q}^* \) decreases, the order quality \( N^*_1 \) will increase from Equation (8), respectively. On the other hand, when \( S_c \) increases, we can see that the reorder point \( ROP_{1q}^* \) will increase from Equation (9) and (10); the reorder point \( ROP_{1q}^* \) increase that will make the order quality \( N^*_1 \) decrease.

2. When \( H_c \) increases:

   The result has two situations. For one thing, when \( H_c \) increases, we can see that the reorder point \( ROP_{1q}^* \) decrease from Equation (9) and (10), the reorder point \( ROP_{1q}^* \) decrease that will make the order quality \( N^*_1 \) increase from Equation (8), but the order quality \( N^*_1 \) increases, the reorder point \( ROP_{1q}^* \) will decrease from Equation (9) and (10). On the other hand, when \( H_c \) increases, we can see that \( \text{the order quality } N^*_1 \) will decrease from Equation (8); the order quality \( N^*_1 \) decreases that will make the reorder point \( ROP_{1q}^* \) increase.

3. When \( O_c \) increases:

   When \( O_c \) is increasing, we can see that the order quality \( N^*_1 \) will increase from Equation (8). In other words, the \( O_c \) increases, it has not had an effect on the reorder point \( ROP_{1q}^* \).

**The EC \( _1 \) influenced by the changes of \( S_c, H_c \) and \( O_c \)**

1. When all other parameters hold fixed, the changes of \( S_c \) will influence on \( EC_1 \) as shown in Equation (21):

   \[
   \frac{\partial EC_1}{\partial S_c} = \frac{E(Y_1)}{N_1} \cdot \int_{ROP_{1q}}^{ROP_{1q}^*} (\theta_{1q} - ROP_{1q}) \cdot f(\theta_{1q}) \cdot d\theta_{1q} > 0
   \]

   From Equation (21), when \( S_c \) is increasing, then \( EC_1 \) is also increasing. It means that \( S_c \) and \( EC_1 \) changes in the same direction.

2. When all other parameters hold fixed, the changes of \( H_c \) will influence on \( EC_1 \) as shown in Equation (22):

   \[
   \frac{\partial EC_1}{\partial H_c} = \frac{M_{1q} \cdot N_1}{2} + \int_{ROP_{1q}}^{ROP_{1q}^*} (ROP_{1q} - \theta_{1q}) \cdot f(\theta_{1q}) \cdot d\theta_{1q} > 0
   \]

   From Equation (22), when \( H_c \) is increasing, then \( EC_1 \) is also increasing. It means that \( H_c \) and \( EC_1 \) changes in the same direction.

3. When all other parameters hold fixed, the changes of \( O_c \) will influence on \( EC_1 \) as shown in Equation (23):

   \[
   \frac{\partial EC_1}{\partial O_c} = \frac{E(Y_1)}{N_1} > 0
   \]
From Equation (23), when $O_\epsilon$ is increasing, then $EC_1$ is also increasing. It means that $O_\epsilon$ and $EC_1$ changes in the same direction.

**EXAMPLE ANALYSIS**

The purpose of this study is to minimize the total expected inventory cost, this study collected the daily sales data, which is a one-dimensional data, using a simple K-Means method for the data. And the K-S test is used to get the demand distribution and construct an inventory decision model. By applying numerical analysis method to calculate the optimal order quantity and reorder point, and verify the feasibility of the proposed model.

**Numerical example**

The supermarket located nearby the suburbs, which opens from 10 am to 9 pm. Through the electronic transaction system, it collected the daily data for three months about product $A$. If the ordering cost per order is 160 dollars, the shortage cost per unit is 12 dollars and the seasonal holding cost is 10 dollars. The given lead time is one day, to find out the optimal combination of decision variables in order to make the total expected inventory cost minimization.

**Numerical example analysis**

At first, collecting the daily data for three months about product $A$ as shown in Figure 1. According to the Figure 1, we use statistical software to divide a week into two groups as shown in Tables 10 and 11.

From Table 11, it reveals that most of Monday, Tuesday, Wednesday, Thursday and Friday are classified into the first group, and then Saturday, Sunday into the second group respectively. Hence, we viewed the first group as non-holiday group and the second group as holiday group.

This study also used statistical software to execute the goodness of fit test and explored the demand distribution of non-holiday and holiday, the result shows the normal distribution is the most appropriate than the other distributions. The related tests shown in Figures 2 and 3.

From Figures 2 and 3, we can see that given $\alpha = 0.05$ and two-tailed test, the K-S value of non-holidays and holiday are $d_{66}^{10} = 0.03053$ and $d_{66}^{\alpha} = 0.05792$. However, the K-S critical values of non-holiday and holiday are $D_{(66, 0.05)^-} = 1.36$ and $D_{(66, 0.05)^-} = 0.17$, it indicates that the result cannot reject $H_0$, so the quantity of sales of non-holiday and holiday follow normal distribution, respectively.

Therefore, under lead time is one day, the demand of non-holiday and holiday are:

The demand of non-holiday $\sim N(115.95, 17.60^2)$

The demand of holidays $\sim N(247.54, 28.20^2)$

**The solution procedure of optimal order quantity and reorder points**

Through by analysis the data, the total expected demand of a given season is 13,973 units, where for non-holiday and holiday are 7537 units and 6436 units, respectively. The parameters are summarized and shown in Table 12.

Given lead time is one day, using the above Equation (8), (9) and (10) by computer programs and numerical computing method to find out the solution of three equations and the optimal combination of $(N_{10}^*, ROP_{10}^*, ROP_{11}^*)$, the related result shows as follows:

$$N_{10}^* = \left[ \frac{1}{K} \left[ \sum_{i=1}^{n} (S_i \cdot q_i) - \sum_{i=1}^{n} (q_i - ROP_i) \cdot f(q_i) \cdot d \cdot q_i) \right] \right]^{1/2} = 674 \text{(units)}$$

$$ROP_{10}^* = F\left[ \frac{S_1 \cdot E(Y_{10})}{M_{10} \cdot H_{10} \cdot E(Y_{10})} \right] = 145 \text{(units)}$$

$$ROP_{11}^* = F\left[ \frac{S_1 \cdot E(Y_{11})}{M_{11} \cdot H_{11} \cdot E(Y_{11})} \right] = 304 \text{(units)}$$

And then we take the second partial derivatives by using the above Equation (11) to Equation (16) as shown in Table 13.

From Table 2, all of the calculated result is greater or equal to zero, so $EC_1$ is a convex function of $(N_{10}^*, ROP_{10}^*, ROP_{11}^*)$, then the Hessian matrix $H$ is verified that $EC_1$ has a minimum value of optimal decision variable, from Equation (17):

$$H(N_{10}^*, ROP_{10}^*, ROP_{11}^*) = \begin{bmatrix} \frac{\partial^2 EC_1}{\partial N_{10}^*} & \frac{\partial^2 EC_1}{\partial ROP_{10}^*} & \frac{\partial^2 EC_1}{\partial ROP_{11}^*} \\ \frac{\partial^2 EC_1}{\partial N_{10}^*} & \frac{\partial^2 EC_1}{\partial ROP_{10}^*} & \frac{\partial^2 EC_1}{\partial ROP_{11}^*} \\ \frac{\partial^2 EC_1}{\partial N_{10}^*} & \frac{\partial^2 EC_1}{\partial ROP_{10}^*} & \frac{\partial^2 EC_1}{\partial ROP_{11}^*} \end{bmatrix}$$
Because the Hessian matrix is the determinant of 3-by-3 and in order to verify these solved values, from Equation (17) that any determinant can be calculated by using expansion, namely:

$$
\left| H \right| = 0.0075 > 0 \cdot \left| H \right| = 0.0061 > 0 \quad \text{and} \quad \left| H \right| = 0.0013 > 0
$$

Then the total expected inventory cost of $EC_1$ is:

$$
EC_1 = \left[ \frac{E(Y_n)}{N_i} \cdot O_i + M_{\infty} \cdot \left( \frac{N_i}{2} \cdot H_i + \int_{0}^{\infty} \left( \theta_{1i} - ROP_{10} \right) \cdot f(\theta_{1i}) \cdot d\theta_{1i} \right) \right]
+ \frac{E(Y_{10})}{N_i} \cdot \int_{0}^{\infty} \left( \theta_{1i} - ROP_{10} \right) \cdot S \cdot f(\theta_{1i}) \cdot d\theta_{1i}
+ \frac{E(Y_{11})}{N_i} \cdot \int_{0}^{\infty} \left( \theta_{1i} - ROP_{11} \right) \cdot S \cdot f(\theta_{1i}) \cdot d\theta_{1i}
= 7165 \text{ (dollars)}
$$

**Sensitivity analysis**

1. When all other parameters hold fixed, the changes of $S_c$ will influence on $N_i^*, ROP_{10}^*, ROP_{11}^*$ and $EC_1$ as shown in Table 14. From Table 14, when $S_c$ is increasing, then $EC_1$ is also increasing. It means that $S_c$ and $EC_1$ changes in the same direction. It confirms the inference of Equation (21).

2. When all other parameters hold fixed, the changes of $H_c$ influences on $N_i^*, ROP_{10}^*, ROP_{11}^*$ and $EC_1$ as shown in Table 15. From Table 15, when $H_c$ is increasing, then $EC_1$ is also increasing. It means that $H_c$ and $EC_1$ changes in the same direction. It confirms the inference of Equation (22).

3. When all other parameters hold fixed, the changes of $O_c$ influences on $N_i^*, ROP_{10}^*, ROP_{11}^*$ and $EC_1$ as shown in Table 16.

From Table 16, when $O_c$ is increasing, then $EC_1$ is also increasing. It means that $O_c$ and $EC_1$ changes in the same direction. It confirms the inference of Equation (23).

**CONCLUSIONS**

This study discussed that lead time is less than one week, because the demand of non-holiday and holiday are different, so the different lead time will influence the quantity of reorder point for each day in a week. We collected daily transaction data to get the demand distribution of non-holiday and holiday under given lead time through by K-Means method and statistical analysis method, and then build up an inventory decision model for heterogeneous demand distribution. Applying numerical analysis method to obtain a set of the optimal decisions, and the Hessian matrix is used to verify the feasibility of the optimal value. Five specific conclusions of this study are drawn as follows:

1. One week is divided into non-holiday and holiday, when the lead time is less than seven days and across holiday and non-holiday situation that will influence the reorder point.
2. The transaction data will be clustered and conducted goodness of fit tests, it can find out the demand distribution of non-holiday and holiday, and through by the distribution with additive to get the demand distribution of lead time. By using numerical analysis method to solve the optimal order quantity and reorder point.
3. When all other parameters hold fixed, the increasing of $S_c$ will raise $EC_1$, $N_i^*$, $ROP_{10}^*$ and $ROP_{11}^*$. Hence, they all changed in the same direction. This result is consistent with Equation (21), and can be verified in Table 14.
4. When all other parameters hold fixed, the increasing of $H_c$ will raise $EC_1$. However, the increase of $H_c$ will reduce $N_i^*$, $ROP_{10}^*$ and $ROP_{11}^*$. This result is consistent with Equation (22), and can be verified in Table 15.
5. When all other parameters hold fixed, the increasing of $O_c$ will raise $EC_1$ and $Q_c$. However, the increase of $O_c$ will not influence on $ROP_{10}^*$ and $ROP_{11}^*$. This result is consistent with Equation (23), and can be verified in Table 16.

**REFERENCES**


Boyacioglu H, Boyacioglu H (2008). Water pollution source assessment by

Cite this article as:
Submit your manuscript at http://www.academiapublishing.org/journals/jbem