Optimal road project portfolio selection problem with local budget constraints

Accepted 11th October, 2019

ABSTRACT

In most cases of road project portfolio selection problems, a best subset of a large set of possible road project is selected subject to an overall limited budget. Most formulations of budget allocation to projects have the general form and structure of a classical 0-1 knapsack problem, and since knapsack problems have non-polynomial complexity, the proposed traditional formulation is NP-hard, the non-polynomial complexity of existing formulations is prohibitive and dictates the use of search techniques for solutions. This study tackles a special case of the road project portfolio selection problem in which each local authority has a specific budget; also, each road project can be financed by budgets originated only from the two local authorities linked by it. The technique presented in this study (and illustrated by a numeric example) uses a transfiguration of the problem into a minimum cost network flow problem - which ensures optimal solution in a polynomial time.

Key words: Transportation, project selection, maximal flow optimization, minimum cost network flow, knapsack problem.

INTRODUCTION

Government transportation agencies are faced with the problem of efficiently selecting a subset of road construction projects for implementation. The road project portfolio selection problem has been considered by a large number of studies (e.g., Yang and Bell, 1998; Drezner and Wesolowsky, 2003). There is a broad variety of mathematical programming models having a wide range of assumptions regarding the required budget (or other) resources, the costs and estimated utilities of the road projects, as well as the overall budget. Each model is suitable to a specific type of cases.

In most road project selection problems described in the literature, a single overall budget is assumed, and partly or wholly allocated to the potential road projects. This problem is often described as a version of the knapsack problem. This study addresses a specific case of the general road selection problem. Each potential road project is running between two different local authorities (or cities, or regions, or boroughs) that directly benefit from its construction, and so may share the budget required to its construction. Each road project can be either constructed (or not) at a given cost. Each local authority carries a budget which can be allocated only to road projects that connect it with other local authorities. Thus, a cost of a specific road project can be budgeted only by the two local authorities connected by it – although the budget does not have to be evenly allocated by them. The objective of the problem is to allocate as much as possible the given budget to the projects, according to the budget constraints of each local authority. The problem of road projects selection, with local government budget constraints, can be associated with wider topic of transport decisions in a multi-level government world (see, for example, Besley and Coate, 2003).

Three forms of road projects may be considered in a portfolio of potential road construction project:
independence, complementarity and substitution (Tzeng and Teng 1993). Complementary projects can enhance performance of overall objectives, while substitutive projects can only substitute, but not increase objective performance. In this study, it is assumed that all the potential road projects have independent costs and independent benefits, thus the overall costs of all selected road projects, and their overall benefits, are equal to the aggregated costs of the selected road projects and their aggregated benefits, accordingly. The problem presented in this study can be extended to include complementarity and substitution forms of road projects, but such an extension is not in the scope of the present study.

Most formulations of budget allocation to projects have the general form and structure of a classical 0-1 knapsack problem, and since knapsack problems have non-polynomial complexity (Kellerer et al., 2004; Gary and Johnson, 1979), the proposed traditional formulation is NP-hard (Archetti et al., 2010). In specific, the 0-1 knapsack problem is also NP-hard (Martello et al., 2000). Moreover, the budget and costs are connected in the constraints based on somewhat artificial assumption on evenly divided budget pre-dedicated to each possible emanating road project. The budget-cost network model, described in the next section, though simpler, treats complexity and budget-cost relationships better, and its solution is obtained through the well-known Minimum Cost Network Flow (MCNF) model (Klein, 1967).

**MODEL DESCRIPTION**

**The original formulation of the road project selection problem with local budget constraints**

Original project selection nomenclature:

- $i, j$ indexes of local authorities connected by a potential road project (end points) (where $n$ is the total number of local authorities)
- $B_i$ The budget of local authority $i$ for new road projects
- $C_{ij}$ The cost of potential road project between two local authorities: $i-j$
- $Y_{ij}$ The percentage of cost of road segment $ij$ borne by local authority $i$
- $X_{ij}$ Zero-One (0/1) variable denoting selection of road project $i-j$

The objective of the selection problem is to maximize the allocation of the available budgets to the construction of new road projects. The budget of a local authority $i$ is given in terms of monetary value ($B_i$). The construction costs of a road project between two local authorities $i$ and $j$ are also given in terms of monetary value ($C_{ij}$). It is assumed that all projects can be justified independently (for example, their benefit-cost ratio is larger than 1). The benefits associated with each road project are not formulated in the problem presented here (this would be an obvious extension of the problem, left for future research). A road project is financed by the two local authorities on the road’s endpoints. However, it is not assumed that the local authorities divide their budgets evenly between their emanating road projects. Thus, the problem on hand is not only to select road projects, but to make sure that each local authority budget $B_i$ is allocated only to roads leading to or from the local authority $i$. Note that relaxing the problem to one in which selecting the maximal value of road projects subject to limited total budget is simply a version of the knapsack optimization problem (which in itself is NP-hard). The original problem is formulated below and hereafter is referred as $P1$:

\[
\text{Max } \sum_i \sum_j C_{ij} X_{ij} \tag{1}
\]

s.t.

\[
\sum_j C_{ij} Y_{ij} \leq B_i \quad \forall i \tag{2}
\]

\[
(Y_{ij} + Y_{ji}) - X_{ij} = 0 \quad \forall (i,j) \tag{3}
\]

\[
Y_{ij} \leq X_{ij} \quad \forall (i,j) \tag{4}
\]

\[
X_{ij} \in (0,1) \quad \forall (i,j) \quad X_{ij} \text{ is binary} \tag{5}
\]

\[
0 \leq Y_{ij} \quad \forall (i,j) \tag{6}
\]

While constraint (2) addresses the total local authorities' budget, constraint (3) ensures that if a road project is selected, it will be fully budgeted by the two local authorities linked by it, and if not selected zero budget is allocated. Constraint (4) ensures that $Y_{ij}$ is not greater than $X_{ij}$; constraint (5) makes sure that $X_{ij}$ is binary; and constraint (6) make sure that all $Y_{ij}$ are non-negative.

**Minimum cost network flow (MCNF) model formulation**

In this subsection, we deal with the same problem dealt with, previously. In the previous section, the road project is financed by the two local authorities on the road’s endpoints; however, it is not assumed that the local authorities evenly divide their budgets between their emanating road projects. We shall now list the steps that lead from the formulation of $P1$ to the network flow formulation. This formulation transformation will be termed transfiguration.
**The transfiguration**

1. Start with a physical network in which each local budget is denoted by $B_i$ and the construction cost of each road connecting local authorities $ij$ is denoted by $C_{ij}$.
2. Add a start node ($s$).
3. Add nodes ($i$) connected to "s" for each local authority, assign each corresponding arc the capacity of $B_i$.
4. Add a node ($j$) for each potential road project.
5. Connect each node $j$ to the corresponding pair of local authorities. Assign these two arcs infinite capacity.
6. Make sure your graph is a bi-partite graph where nodes ($i$) of local authorities are on one side of the graph, and nodes ($j$) for potential road projects are on the other side of the graph.
7. Add an end node ($t$) and connect it to the road project nodes, assign each corresponding arc the capacity of $C_j$.
8. Add the arc $X_{st}$ with infinite capacity (as required by the MCNF technique).

The following network flow optimization model is based on the above transfiguration.

**MCNF nomenclature**

$N$ number of local authorities  
$m$ number of road projects ($m \leq n(n-1)$)  
$i$ index 1..nof a local authority (possibly connected to other authorities by potential road projects)  
$j$ index 1..nof a potential road project connecting two local authorities.  
$s, t$ indexes of starting ($s$) and ending ($t$) dummy nodes of a flow network used on the transfigured problem  
$B_i$: The budget of local authority $i$ for new road construction  
$d$: The number of candidate road projects that coincide with (connect to) local authority $i$  
$C_j$: The cost of potential road project $j$.  
$x_{di}$ Budget expenditure of local authority $i$  
$x_{dj}$ Budget allocation by local authority $i$ to road project $j$  
$z_{ij}$ Zero/one (0/1) decision variable indicating the selection or rejection of road project $j$

To transform all local authorities’ budgets, to arc representation, an "s" node is added, and is linked by arcs to all the nodes of the local authority budgets. In a similar way, a 't' node is added, and arcs from all road projects to 't' represent the costs of each road project. The proposed network flow model $P2$ is formulated as follows:

$$\sum_j x_{jt} - x_{ts} = 0 \quad (8)$$

$$x_{si} \leq B_i \quad \forall i \quad (9)$$

$$x_{si} = \sum_j x_{ij} \quad \forall i \quad (10)$$

$$x_{jt} = \sum_j x_{ij} \forall j \quad (11)$$

$$x_{jt} \leq C_j \quad \forall j \quad (12)$$

$$x_{jt} - M(z_{ij}) \leq 0 \quad \forall j \quad (13)$$

$$z_{ij} (1-z_{ij}) = 0 \quad \forall j \quad (14)$$

$$x_{si} \geq 0 \quad \forall i; \quad x_{jt} \geq 0 \quad \forall i, j; \quad x_{jt} \geq 0 \quad \forall j \quad (15)$$

Constraint (8) ensures the balance of flows entering and leaving node $t$. Constraints (9) verify that local expenditures do not exceed the local budget. Constraints (10) are flow balance equations ensuring that all budget expenditures are dedicated to projects. Constraints (11) are summarizing the project usage of funds. Constraints (12) verify that the project expenditures are less than the project cost. Constraints (13) verify that if project $j$ is not selected the corresponding flow would be zero. Constraints (14) ensure that $z_{ij}$ = 0/1. However, the constraints are quadratic rather than integer and therefore the complexity remains polynomial (Brucker, 1984; Billionnet and Calmels, 1996) for issues regarding the solution of quadratic knapsack problems. Constraints (11) ensure that all flows are positive.

The above formulation maximizes the overall allocation of budget to the local authorities, from an overall perspective of an integrated planning entity, while meeting the local constraints – taking into account that all roads emanating from each local authority $i$ would not exceed its budget.

**A NUMERIC EXAMPLE**

**The budget-cost network model**

The budget-cost network model is simple: it involves only budgets (on nodes) and costs (on arcs). The budget-cost network model starts with modeling the physical network as a network of edges (or arcs) and vertices (or nodes): $\{E, V\}$ where local authority construction and maintenance costs are attached to edges $E$ and the budgets are attached to vertices $V$. An example of network with the local authorities’ budgets and the road project costs is shown in Figure 1.
Transformation of the network into a network flow problem

We now show how this problem could be translated into an equivalent network flow problem. The vertices (local authorities) are arranged on the left side of a bipartite graph and the edges (road projects) are arranged on the right side of the bipartite graph as shown in Figure 2. The next step is to draw arcs between the vertices and their corresponding edges (the potential road projects that have the node as one of its end-points). Finally, the budget limit of each local authority is allocated to the vertices, and the road project’s costs are allocated on the edges side. An example of the bipartite graph is shown in Figure 2.

The second stage of the transformation is connecting the two sides of the bipartite graph to a source node $s$ and a sink node $t$. The arcs between the vertices and edges are assigned infinite capacity. The corresponding example continues in Figure 3.

Figure 3 shows the budgets as sources of flow on the left side, the money flows to the right side to pay for the construction costs of the various road projects. The overall objective is to maximize the flow. This is easily measured as max flow on the link $s - t$ (Ford and Fulkerson, 2015). This problem is known to have polynomial time solution (Ervolina and McCormick, 1993). This could be easily verified since the problem’s constraints are based on the node-arc incidence matrix. The optimal solution for the maximal flow problem is depicted in Figure 4.
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Figure 4 shows that the costs of road projects (1,2), (1,3) and (2,3) are fully covered by the budgets, while road project (3,4) is not fully covered, and therefore cannot be executed. Figure 4 also shows that local authority 1 spends 8 out of budget of 9, the budgets of local authorities 2 and 3 are fully utilized. Since no new roads are allocated to local authority 4 it spends none of its budget. The dotted arrows (in Figures 4 and 5) represent zero budget flow while hatched arrows represent partial allocation. Figure 5 shows the road project selection and budget usage implied by Figure 4.

**Sensitivity analysis**

Converting the network flow model presented in Figure 3 into a model is a straight forward task. The variables are the flows on the arcs; their indexes are formed by a combination of the origin-destination node indexes. For example, the arc from t to s is annotated X_{st}. The model maximizes the use of the overall budget, X_{st}, and includes three types of constraints: the initial budget constraints of each local authority, the flow balance constraints, and the costs constraints of each road project. Thus, the following model represents the formulation corresponding to Figure 3 and fed into a solver:

Max X_{st}
Subject to
X_{v1}<9
X_{v2}<3
X_{v3}<6
X_{v4}<6
X_{v1}X_{v1v12}X_{v1v13}=0
Therefore, the solution is based on a Max Flow formulation, which is strictly polynomial. This formulation could be extended to the minimum cost network flow problem (MCNF) which is also polynomial time algorithm.

**CONCLUSIONS**

This study identifies a special version of the road project portfolio selection problem; road projects on the network are financed by budgets of local authorities rather than by an overall budget. A road project can be financed only by the specific local authorities connected by it. The study presents a model based on local authority budgets and project costs, along with its conversion process of it into a maximal flow optimization model (and in specific – a Minimum Cost Network Flow model). Since the maximal flow problem is known to have polynomial time solutions – this formulation has polynomial time complexity. As far as we know, no other study has identified the specific problem presented in the present study, neither the solution described in it. The main novelty of the present study is the transformation of the aforementioned selection problem with local authorities’ budget constraints into a polynomial flow problem.

Several extensions of the model can be considered: (1) addressing a more general problem, where road projects may be linked to (and budgeted by) any number of local authorities (and not just 2); (2) addressing the benefits, and not only the costs, associated with each road project and local authority; (3) addressing complementarity and substitution forms of road projects (Tzeng and Teng, 1993); (4) incorporating aspects of fairness in the allocation of budgets between two local authorities; (5) addressing the multi-criteria nature of the transportation project portfolio problem (Tzeng and Teng, 1993; Current and Min, 1986); and (6) addressing the fuzzy nature of costs, benefits and budgets in the transportation project portfolio problem (Tzeng and Teng, 1993; Avineri et al., 2000).

**REFERENCES**


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