ABSTRACT

This study presents a methodology to detect significant changes in the price of U.S. dollar in terms of stock exchange using the Haar wavelet as well as statistical analyzes of several parameters. The data used are based information provided by the stock market, both for the U.S. dollar and for the stock market. The results show that the methodology used through Haar wavelet are able to inform in advance about the trend of the stock market through a single variable, the U.S. dollar. We would like to point out that we selected the period of June 2008 because it is a time when we had large variations from US dollar and Bovespa index.

Key words: Stock market, dollar, Haar wavelet.

INTRODUCTION

The financial market, at large, has become the target of intense research in the past few years. Increasingly, some tools from engineering and computing, once used only in industrial processes, are being widely applied in the financial market in order to optimize stock portfolio (Mohamed et al., 2009), and predict the stock market (Nair, 2010). Besides these objectives, the research of correlation and dependence among variables of the financial market is also a much-explored field.

An area of study, which deserves attention in the financial market, is the dynamic behavior of the dollar, since this currency influences the behavior of the market. The contact of Brazil with The United States has narrowed in recent years, which classifies the United States as one of Brazil's biggest commercial partners. In this sense, the dollar variations affect directly the Brazilian economy, that is, it has a direct influence on the currency exchange, causing several reactions concerning import and export. Given the importance to the subject, many researches have been developed in order to analyze the dollar variation in relation to some variables: in a study by Coudert et al. (2008), through cointegration and causality tests, the relationship between dollar and the oil price was investigated; Richards and Simpson (2009) examined, through models based on Vector Autoregression (VAR), the relationship between the stock prices and the Australian dollar; while Agrawal et al. (2010) studied the dynamic behavior of the dollar and the volatility of the stock market through statistical tests.

Finally, Rahman and Uddin (2008) investigated the existing relationship between the stock prices and the exchange rate. Such researches instigated the proposal of a new methodology based on the Haar wavelets and the statistical analysis of the details coefficients produced by time series analysis, allowing the detection of meaningful changes in the exchange rate in relation to Brazilian stock market.

The main objectives of this study were to investigate the dynamic behavior between the exchange rate and the stock market, and determine the important information that is required by investors, as well as to Brazilian economy. Thus, a new methodology was introduced here which, by using Haar wavelets and the statistical analysis of the details coefficients produced by time series, allows to detect a dynamic relationship between US dollar and Brazilian stock market and what may be useful as a complementary tool for the investors and public companies.
WAVELETS

Wavelets are functions capable of decomposing and representing another hierarchically function by allowing a function to be described, roughly, in terms, afterwards in other forms which present, gradually, greater details, enabling the function analysis in different scales of frequency and time.

The analysis through wavelets has been a promising alternative substituting the classical analyses, which use Fourier series, mainly in the treatment of acoustic signals, in the interpreting of seismic signals and in the solution of numerical methods applied in several other areas:

$$\psi_{a,b}(x) = |a|^{-\frac{1}{2}} \psi \left( \frac{x-b}{a} \right) \quad a,b \in \mathbb{R}, a \neq 0$$  

(1)

Wavelet algorithms process data in different scales or resolutions, which provide the decomposition of a function in other functions, becoming useful in the sense that the signal characteristics may be more noticeable and better explored. By analogy, looking up a signal through a large window, brutal characteristics are noticed, whereas if the window is small, richer details of signal characteristics are observed.

Thus, being possible to divide a signal into intervals and, further, subdivide into other intervals, it is possible to extract more refined characteristics of the analyzed signal. With the wavelets, it is possible to execute these divisions, to refine and expose detailed characteristics of the signal being studied.

The analysis through wavelets adopts a prototype function, called wavelet mother and time analysis and frequency are made through contraction and dilation of the studied function. To obtain the data in different resolutions, the wavelet analysis is used in scale equation (sizing), as shown in Equation 2:

$$W(x) = \sum_{k=-N/2}^{N/2} (-1)^k c_{k+1} \psi(2x + k)$$

(2)

Where $W(x)$ is the sizing function for a mother function, $\psi$, $c_k$ are the wavelet coefficients. The wavelet coefficients can be seen as a filter and are placed in a matrix of transformation, which is, applied a vector of data (a signal). These coefficients are ordered in a way that some of them act to refine the signal and others so as to evidence the detailed information of the signal.

Absolutely, the coefficient matrix is applied in a vector of data as follows: the coefficients are arranged in matrix as previously described and the matrix is applied, first, in the original vector of data. Afterwards, the vector is refined and divided in half and the matrix is applied again. This vector, once divided, is refined and divided again and the coefficient matrix is applied. This process is continued up to a trivial number of refined data remains. In other words, each application of the coefficient matrix makes the resolution higher, that is, in each application, the minimum details of the signal characteristics are evidenced.

There are several types of wavelets capable of building other family models which adapt in a more suitable way in each case (Morettn, 1997). In Figure 1, for example, it is represented the Morlet wavelet or Gaussian modulation through Equation 3:

$$\psi(x) = e^{-i\omega x} e^{-x^2/2}$$

(3)

The wavelet known as Mexican hat, represented in Figure 2, is shown in Equation 4:

$$\psi(x) = (1-x^2)e^{-x^2/2}$$

(4)

In Figure 3, the Shannon wavelet is shown, according to Equation 5:

$$\psi(x) = \begin{cases} \sin(\frac{\pi x}{2}) & 0 < x \leq 0.5, \ \cos(\frac{3\pi x}{2}) & \text{to other intervals} \\ \frac{\sin(\pi x)}{\pi x} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

(5)

Wavelets can be applied in numerous purposes, namely computational vision, fingerprints compression, recovery of signals, similarity detection in a time series, etc. The fundamental objective of the present study is to investigate the existing similarity or dissimilarity between the exchange rate, in terms of the US dollar, and the stock market, which will be represented by the stock market index of the State of São Paulo – Ibovespa. For this purpose, it is proposed to use a type of wavelets known as Haar wavelets, which can use several types of expansion functions such as the pulse function, truncated cosine, etc. (Newland, 1993).

The Haar wavelets are composed of a function called mother and another called scale, represented, respectively, in Equations 6 and 7. The combinations that allow applications in two dimensions, equated according to Aboufad and Schlifer (1999), are shown in Equation 8:

$$\psi_{j,k}^{(H)}(x) = 2^{j/2} \psi \left( 2^j x - k \right) \quad j,k \in \mathbb{Z}$$

(6)

$$\psi^{(H)}(x) = \begin{cases} 1 & 0 \leq x < 0.5, e \\ 0 & \text{to other intervals} \end{cases}$$

(7)

$$\psi_{j,k}^{(H)}(x) = \left[ \phi(x) \psi(x) \psi(2x) \psi(2x-1) \cdots \psi(2^j x-k) \right]$$

$$\psi_{j,k}^{(H)}(y) = \left[ \phi(y) \psi(y) \psi(2y) \psi(2y-1) \cdots \psi(2^j y-k) \right]$$

$$\left\{ \psi_{j,k}^{(H)}(x,y) \right\} = \left[ \phi(x) \phi(y) \phi(2x,2y) \phi(2x-1,2y-1) \cdots \right]$$

(8)
It can be observe that these equations are indexed by two parameters “j” and “k” which are responsible for the accuracy of the results through several levels of resolution.

**APPLICATION**

The data used in this study are the daily oscillations of Bovespa index prices and the US dollar during a period ranging from January 2nd, 2008 to June 30th, 2008. These data are from Bovespa site and Banco Central do Brasil site.

The methodology which is used here comprised the standard deviation analysis, correlation, covariance of maximum, minimum and the detail coefficients of the samples after the application of Haar wavelets, and also the remains and the spectrum of the Caetano obtained signals (2008). Thus, a group of one hundred and twenty two values, either the US dollar or the stock quotes, in the period ranging from January 2nd, 2008 to June 30th, 2008, are obtained, as shown in Table 1.

The Equation 9 shows that the sample linear trend estimated using the minimum square method, having \( t \) as the number of days, according to Caetano and Yoneyama (2007):

\[
y_L(t) = 89.944t + 58867 \quad (9)
\]

Figure 4 shows the values with a linear trend. Despite the possibility of the wavelets application in a sample with linear and cyclic trend, we may be able to add extra coefficients, which could harm the analysis, such as noises. After the removal of the linear part \( y_L \), the series \( y_R = y - y_L \) presents a component with cyclic trend.

Through the minimum square method, which is a technique that tries to find the best adjustment for a group of data, minimizing the sum of the squares of the differences between the estimated value and the observed data, it is possible to make an estimate of the cyclic

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**Table 1: Part of the data of the US dollar and Bovespa index.**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Day (d/m/y)</th>
<th>USD</th>
<th>IBOV(Points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2/1/2008</td>
<td>1.7722</td>
<td>62815</td>
</tr>
<tr>
<td>2</td>
<td>3/1/2008</td>
<td>1.7569</td>
<td>62891</td>
</tr>
<tr>
<td>3</td>
<td>4/1/2008</td>
<td>1.7574</td>
<td>61036</td>
</tr>
<tr>
<td>4</td>
<td>7/1/2008</td>
<td>1.7675</td>
<td>60772</td>
</tr>
<tr>
<td>5</td>
<td>8/1/2008</td>
<td>1.7554</td>
<td>62080</td>
</tr>
<tr>
<td>6</td>
<td>9/1/2008</td>
<td>1.7691</td>
<td>62673</td>
</tr>
<tr>
<td>7</td>
<td>10/1/2008</td>
<td>1.7628</td>
<td>63515</td>
</tr>
<tr>
<td>8</td>
<td>11/1/2008</td>
<td>1.7536</td>
<td>61942</td>
</tr>
<tr>
<td>13</td>
<td>18/1/2008</td>
<td>1.7850</td>
<td>57506</td>
</tr>
<tr>
<td>14</td>
<td>21/1/2008</td>
<td>1.8301</td>
<td>53709</td>
</tr>
<tr>
<td>15</td>
<td>22/1/2008</td>
<td>1.8041</td>
<td>56097</td>
</tr>
<tr>
<td>16</td>
<td>23/1/2008</td>
<td>1.8147</td>
<td>54234</td>
</tr>
<tr>
<td>17</td>
<td>24/1/2008</td>
<td>1.7883</td>
<td>57463</td>
</tr>
<tr>
<td>18</td>
<td>28/1/2008</td>
<td>1.7906</td>
<td>58593</td>
</tr>
<tr>
<td>19</td>
<td>29/1/2008</td>
<td>1.7762</td>
<td>59529</td>
</tr>
<tr>
<td>20</td>
<td>30/1/2008</td>
<td>1.7802</td>
<td>60289</td>
</tr>
</tbody>
</table>
component, having the variable $t$ as the number of days, according to Equation 10 (Gilat and Subramaniam, 2008; Ruggiero and Lopes, 1996; Tenkolsky, 1992):

$$y_C(t) = 1985.65 * \cos(-0.07t)$$

(10)

The Figure 5 shows the series $y_R$ and $y_C$ of the values with a linear trend. After the $y_C$ removal, the series $y_F = y_R - y_C$ is obtained. This series $y_F$ does not either have a low frequency cyclic component or a linear trend, as shown in Figure 6.

Table 2 shows the statistical values of the group, either the US dollar or the stock quotes from January 1st, 2008 to June 30th, 2008.

Table 2 and Figure 7 show the existence of a negative correlation between US dollar and stock market index, that
Figure 6: Original sample with linear and cyclic trend removed.

Table 2: Correlation, covariance, standard deviation, USD and Ibovespa maximum and minimum.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>IBOV</th>
<th>USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>73,516.00</td>
<td>1.8301</td>
</tr>
<tr>
<td>Minimum</td>
<td>53,709.00</td>
<td>1.5919</td>
</tr>
<tr>
<td>Average</td>
<td>63,612.50</td>
<td>1.7110</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>4,338.81</td>
<td>0.0548</td>
</tr>
<tr>
<td>Sd deviation (%)</td>
<td>6.82</td>
<td>3.20</td>
</tr>
<tr>
<td>Covariance</td>
<td>-177.86</td>
<td></td>
</tr>
<tr>
<td>Correlation (%)</td>
<td>-75.42</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7: Variation of the US dollar in relation to the stock market index.
Table 3: Statistical values after the application of the Haar wavelets.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>IBOV</th>
<th>USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>72,764.91</td>
<td>1.8310</td>
</tr>
<tr>
<td>Minimum</td>
<td>53,488.00</td>
<td>1.6027</td>
</tr>
<tr>
<td>Average</td>
<td>63,126.45</td>
<td>1.7169</td>
</tr>
<tr>
<td>Standard deviation (%)</td>
<td>6.77</td>
<td>3.18</td>
</tr>
<tr>
<td>Covariance</td>
<td>-174.32</td>
<td></td>
</tr>
<tr>
<td>Correlation (%)</td>
<td>-75.42</td>
<td></td>
</tr>
</tbody>
</table>

is, when the US dollar goes up and stock market index goes down and vice versa.

For the dollar and Ibovespa values, the Haar wavelets are applied using the resolution level 6, generating the coefficient numerical values for the US dollar and Ibovespa (Chapman, 2009; Hanselman, 2008).

Table 3 shows the statistical values obtained of correlation, covariance, standard deviation, the maximum and minimum of the US dollar and stock market index after the application of the Haar wavelets, according to Brandimarte (2009) and Tong and Lee (2009).

The application of the Haar wavelets in the historical data of US dollar and Ibovespa results of the signals are shown in Figure 8. It can be observed that after the application of the Haar wavelets in the used data, the dollar price and Ibovespa keep presenting a negative correlation, making one believe that there is, between the US dollar and Ibovespa, an inverse reaction. In other words, when dollar goes up, Ibovespa goes down and vice-versa.

The results so far show that the Haar wavelets are capable enough of, besides filtering data by eliminating possible noises (discrepant values in historical series), displaying the real exchange rate trend in relation to Ibovespa. As a result, it is intended from that point, to investigate the dynamic dollar behavior in the determined subperiod of a sample using the Haar wavelets and, therefore, propose a model to foresee possible behaviors of the US dollar.

In Figure 9, we can be observed a graph of the detail coefficients of the Haar wavelets transform considering the USD variation in relation to Ibovespa. The original signal removing itself from the cyclic tendencies can be seen at the upper part of the same figure.

The whitish region, where the blue line goes, shows the existence of the coefficients with great magnitude. We see that the region domain of high frequencies is represented by a “flash” in white which might be able to indicate a strong trend of value inversion or a critical moment. Thus, this blue line shows the results before and after the application of wavelets.

Figure 10 shows the US dollar dynamics, represented by six hundred and twenty seven samples, from January 2nd, 2008 to July 1st, 2010 (Pazza, 2009; Correia, 2008).

For purposes of analysis, some changing USD trajectory points is shown in Figure 11. The samples highlighted in Figure 11 show, clearly, the following characteristics:

a) Sample 1 has a high trend reversed to a low trend.
Figure 9: Response of the application of Haar wavelets on the series yF.

![Haar Wavelets Application](image)

Figure 10: Dynamic behavior of USD.

b) Sample 2 has a low trend reversed to a high trend.
c) Sample 3 has a low trend reversed to a high trend.
d) Sample 4 has a high trend reversed to a low trend.

By applying the Haar wavelets in each sample of the Figure 11, it is obtained the linear trend and the Haar coefficients trend for each subperiod of the Figure 11. These results are
presented in Figures 12 to 19 which show the linear trend and the Haar coefficients of the four samples.

By observing, for example, the Figure 12, it is possible to verify that the graph which represents an inclined linear trend opposite to original trend, indicates low. The original trend of the sample was high, but after the application of the Haar wavelets, the trend turns to be low.

Table 4 shows a summary of the results of the original trend (OT), the linear function after the application of the Haar wavelets (LFW), which is a trend produced by the proposed model (TM), and the original trend which occurs before the original trend of the samples (OT-).

It can be observed from the results shown in Table 4 that the original trend of the dollar behavior in sample 1 is high, and the original trend which precedes sample 1 is low. Curiously, the model based on the Haar wavelets shows, for the considered sample, a low trend in the dollar price and that occurs for all the samples. In this particular case, it seems that the proposed model is strongly influenced by the heuristics arising from the theory of behavioral
Figure 13: Linear trend of the Haar coefficients from the first sample.

Figure 14: Linear trend from the second sample.

Figure 15: Linear trend of the Haar coefficients from the second sample.
Figure 16: Linear trend from the third sample.

Figure 17: Linear trend of the Haar coefficients from the third sample.

Figure 18: Linear trend from the fourth sample.
finances. This suggests the effects of overreaction and underreaction in the model results. In other words, when the obtained results show a dollar low, it means that, actually, the dollar will increase and vice versa. Since the Ibovespa and the dollar have a negative correlation, the proposed model by the Haar wavelets indicates the real trend of Ibovespa.

### CONCLUSION

This study shows the theoretical aspects and a study of the dynamic relationship between the US dollar and the State of São Paulo Stock Market using the Haar wavelets. The proposed methodology allows the determination of the coefficients' numerical values, in function of the level of resolution, and also the analysis of the detailed statistical results.

In the analysis, the minimum square method is included, which allows, through the linear cyclic trend, determination of the parameters and graphs that indicate the sample graph trends analyzed with great accuracy. The results show two important trends:

a) The US dollar and Ibovespa, in all the analyzed period, have a negative correlation, that is, when a dollar increase occurs, Ibovespa displays a fall and vice versa. They seem to have, between dollar and Ibovespa, deeds of overreaction and under reaction arising from the theory of behavioral finances;

b) Another interesting aspect is the fact that, when the methodology is applied based on the Haar wavelets in subperiods of the US dollar’s behavior, the model displays a piece of information always contrary to the dollar trend. Since Ibovespa and dollar have a negative correlation, the proposed model by the Haar wavelets indicates the real Ibovespa trend.

The dynamic behavior of the dollar has a considerable influence on Brazilian economy, as the exchange rate appreciation, which is related to the dollar fall in relation to Brazilian currency, may directly encourage imports and, the exchange rate depreciation, which is related to the Brazilian currency fall in relation to the dollar, may encourage exports. In this sense, the proposed methodology proves to be effective by informing the investor, beforehand, the real dollar trend, as well as the Ibovespa trend.

### REFERENCES


Tong JYC, Lee PV (2009). Bolsa de Valores, Visão Feliz e Otimista de Três Gerações, Age.

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