Study on the Asymmetry Information Problem Based on Principal -Agent Theory

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ABSTRACT

The relationship agent and principal information asymmetry is a typical agent-principal problem. The core of this theory design the participation constraint, individual rationally constraint (IR) and incentive compatibility constraint (IC) and the principal expected unity function is maximization. This study analyzes the principal relationship between agents through establishing a gambling model. The contribution of this study is the formulated model of principal-agent relationship between the agent and the principal. In this model, the present study thoughtful moral hazard is different from most existing research in this field. Finally, the paper gives some suggestions and conclusion provides information on the company that should also consider shared responsibility and professional competence.

Key words: Principal-agent, information asymmetry, incentive compatibility constraint, individual rationally constraint.

INTRODUCTION

Information asymmetry refers to the seller and buyers involved in the transaction that can affect the trading of different information. In general, sellers have more information about trading goods than buyers, but the opposite situation may exist. Examples of the former can be found in the sale of used cars where the seller sells a vehicle to understand the buyer. Examples of the latter such as health insurance, the buyer usually has more information.

Asymmetric information may lead to adverse selection. The principal-agent problem is also known as agency problems and agency dilemma due to different objectives and circumstances arising from conflicts of interest between the principal and the agent. When the agent has some motivation that drives his behavior then, this goal increase the focus on their own interests, rather than increasing the interest of the principal; there will be the phenomenon of this dilemma. When the agent has some motivation that drives his behavior goal is to increase their own interests, rather than increasing the principal's profit, there will be the phenomenon of agency dilemma. For example, in the agency problems there is the company operator (agent) and shareholder (principal), mutual fund manager (agent), investor (principal), politicians (agent) and voters (principal).

The theory of Principal agency is feasible in solving Principal-agent problems. The core of this theory designing participation constraint, individual rationally constraint (IR) and incentive compatibility constraint (IC) and the principal expected unity function is maximization. Under the condition of information asymmetry, optimal sharing principles satisfy Holmstrom- Milgrom condition (Qin, 2004; Holmstrom, 1979). It enables both the principal and agent to realize double win (Guo and Gu, 2006).

Principal-agency theory to analysis of problem of principal-agent can improve agential construction's implementation efficiency (Zhu et al., 2011; Zhang et al., 2015). Xiaojun and Tao (2012) research the problem of project supervisor industry based on game theory. Some researchers like Li et al. (2012) and Li and Zhou (2011) used a venture capital dynamic incentive model in Moral Hazard problems.
THE UNCERTAIN PRINCIPAL-AGENT MODE

This paper uses the following notations:

\( \pi \): The level of effort of supervisor;
\( S() \): The contract between the two parties;
\( B \): The project share of the output;
\( \alpha \): The fixed income of the agent;
\( c(a) \): The agent's cost;
\( B() \): The principle preferences utility function;
\( U() \): The unity function of the agent is with characteristics of risk adverse.

Assumption

The following are assumptions made:

- Assumption
  \( B' > 0, \ B'' \leq 0, \ U' > 0, \ U'' \leq 0, \ C' > 0, \ C'' > 0 \)

1) \( C' > 0 \) It indicates that agent wants their little efforts, \( B'' \leq 0 \) and \( U'' \leq 0 \) indicates diminishing marginal utility.
2) \( C'' > 0 \) indicates increasing marginal costs.
3) The agent level of efforts is a continuous variable. \( \frac{\partial \pi}{\partial a} > 0 \) indicates principal hope that agents has more efforts.
4) The agent work is uncertain. The output is not only determined by own effort, but also by the outside control of the objective conditions.

THE OPTIMIZED MATHEMATICS MODEL

(a) Principal’s optimization

The owner’s expected value utility is equal to expected income given as:

\[
E(B(x - s(x)))
\]

(b) Incentive compatibility constraint (IC)

Assume the unity function of supervisor is with characteristics of risk-adverse, the supervisor’s actual income therefore becomes:

\[
w = s(x) - c(a)
\]

The agent’s expected utility function is equivalent to the certainty equivalent income minus the random cost risk, that is:

\[
E(w) - \text{The random cost of risk} = E(U)
\]

The random cost of risk is \( \frac{1}{2} \rho \Var(S(x)) \)

(c) Participation constraint (IR) given as:

\[
E(U) > E(s(x) - C(a^{-1}))
\]

Participation constraint is redundant, as long as \( s(x) \) is sufficiently small, participation constraint (IR) is satisfied. The Principal’s choice is the following optimization problem given as:

\[
\Max (B(x - s(x)) \ S.t. \ E(w) = E(U) = U(s(x) - c(a))
\]

The aforementioned model is solved using Lagrange function given as:

\[
L(s(x), \lambda) = B(x - s(x)) + \lambda U(s(x) - c(a))
\]

\[
\frac{\partial L}{\partial S} = -B'(x - s(x)) + \lambda U'(s(x)) = 0
\]

Therefore:

\[
\lambda = \frac{B'(x - s'(x))}{U'(s'(x))}
\]

It means that ratio of principle and agent's marginal utility is a constant. This is a typical Pareto optimal state. If the principal and agent are strictly risk averse, that is, \( B'' < 0 \) and \( U'' < 0 \), agent and principal bears the risk. If the principle is risk-neutral, that is, \( B'' = 0 \), agent is strictly risk adverse and is \( U'' = 0 \), principal’s indifference curve is a straight line and agent does not bear all risk but the Principal bears all risk. The Principal’s marginal utility is a constant. Assume \( B' = 1 \). Then:

\[
\lambda = \frac{B'(x - s'(x))}{U'(s'(x))} = \frac{1}{U'(s(x))}
\]

Since \( U' \) is with \( s(x) \) decreasing, \( s(x) \) is constant, that is, agent independent of income and output \( x \). If the principal is strictly risk adverse, that is, \( B'' < 0 \), agent is risk-neutral, that is, \( U'' = 0 \), then, agent’s indifference curve is a straight line; agent bears all risk \( s(x) = x - x^0 \), the
principal get a fixed income \( x - s(x) = \chi^0 \). If the principal and agent are risk-neutral, that is, \( B'' = 0 \) and \( U'' = 0 \), then the principal and agent’s indifference curve is a straight line; So, at any point it is optimal. On both sides of Equation (6) the derivative \( x \) yields:

\[
-B'' \left( 1 - \frac{ds^*}{dx} \right) + \lambda U'' = 0
\]

Substitute \( \lambda = \frac{B^*}{U} \) into the aforementioned Equation yields:

\[
\frac{ds^*}{dx} = \frac{rp}{rp + ra}
\]

(9)

Where, \( rp = -\frac{B''}{B} \), it indicates principal’s measure of absolute risk aversion , \( ra = \frac{-U''}{U} \). It also indicates agent’s measure of absolute risk aversion. The Equation (9) means that the relations agent’s paid and output was entirely decided by the absolute risk aversion measure. When \( ra > 0 \), \( rp > 0 \), it says the agent and principal are the absolute risk aversion measures. \( 0 < \frac{ds^*}{dx} < 1 \), the agent’s payment \( s^* \) rise with the \( x \) rise and rise less than \( x \) rise.

When \( rp = 0 \), \( \frac{ds^*}{dx} = 0 \), then \( s^* \) is constant , it means \( s^* \) has nothing to do with \( x \). When \( ra = 0 \), \( \frac{ds^*}{dx} = 1 \) then, \( s^* \) rise equals the \( x \) rise.

THE ANALYSIS OF PRINCIPAL_AGENT GAME

Assume ‘\( a \)’ is one dimension effort level variable. Assume that the agent’s output function and the level of effort is a linear relationship: \( x = a + \theta; \theta \sim N(0, \sigma^2) \), \( E(x) = E(a + \theta) = a \quad V(x) = V(a + \theta) = \sigma^2 \)

The contract between two parties using a linear Equation is given as:

\[
s(x) = \alpha + \beta x.
\]

Where \( \alpha \) is agent’s fixed income, \( \beta \) is the project share of the output. The principal’s expected value utility is equal to the expected income (the principle is risk-neutral, \( B(x) = x \) that is, \( B'' = 0 \) ) given as:

\[
E(x - s(x)) = E(x - \alpha - \beta x) = -\alpha + (1 - \beta)\alpha
\]

(10)

The \( \beta = 0 \) means that the agent does not take risk; \( \beta = 1 \) means that the agent bears all risk. Assume the unity function of the agent is \( U(x) = -e^{-\rho \xi} \) with characteristics of risk adverse (that is, \( U'' < 0 \)), where \( \rho \) is absolute measure of risk aversion, \( w \) is the real monetary income.

Assume the agent’s cost function is \( c(a) = \frac{1}{2}ya^2 \), where \( \gamma \) represents the cost factor, the larger \( \gamma \) is assigned and the greater negative is bought. The agent’s income is given as:

\[
w = s(x) - c(a) = \alpha + \beta(a + \theta) - \frac{1}{2}ya^2
\]

(11)

\[
E(w) = \alpha + \beta a - \frac{1}{2}ya^2
\]

(12)

Since the agent is risk adverse, the random cost risk is:

\[
\frac{1}{2} \rho \text{Var}(s(x)) = \frac{1}{2} \rho \text{Var}(\alpha + \beta x) = \frac{1}{2} \rho \beta^2 \sigma^2
\]

(13)

\[
E(U) = E(w) - \frac{1}{2} \rho \beta^2 \sigma^2 = \alpha + \beta a - \frac{1}{2}ya^2 - \frac{1}{2} \rho \beta^2 \sigma^2
\]

(14)

(I) The principal can observe the effect level of the agent. The incentive constraint (IC) does not work. The optimized mathematics model is therefore given as:

\[
\max \quad E(B(x - s(x)) = -\alpha + (1 - \beta)\alpha \quad \alpha,\beta,a
\]

(15)

(II) s. t. \( \alpha + \beta a - \frac{1}{2}ya^2 - \frac{1}{2} \rho \beta^2 \sigma^2 \geq U^0 \)

(16)

Where \( U^0 \) is the supervisors retained income level.

Substitute the constraint \( \alpha = -\beta a + \frac{1}{2}ya^2 + \frac{1}{2} \rho \beta^2 \sigma^2 - U^0 \) into objective function yields:

\[
h(a, \beta) = a - \frac{1}{2}ya^2 - \frac{1}{2} \rho \beta^2 \sigma^2 - U^0
\]

(17)

\[
\frac{\partial h}{\partial a} = 1 - ra = 0
\]

(18)
\[
\frac{\partial h}{\partial \beta} = -\rho\beta\sigma^2 = 0
\]

The solution of Equations (18) and (19) is:
\[
a^* = \frac{1}{\gamma}, \quad \beta^* = 0
\]

Substitute the Equation (20) into (16) yields:
\[
a^* = U^0 + \frac{1}{2\gamma}
\]

From the aforementioned statement, we have the following results:

1. The principal is risk-neutral \( B'' = 0 \), this is \( r_p = 0 \) and \( \beta^* = 0 \).
2. Agent’s fixed income \( (\alpha^*) \) that is, principal pays to agent equals to the agent’s retained income \( (U^0) \) plus the cost of effort \( \frac{1}{2\gamma} \).
3. From Equation (20) \( \alpha^* = 1 \), this means that the optimal output level is 1.
4. When the principal observe the agent’s effort \( a < \frac{1}{\gamma} \), the principal will pay for the agent less than \( a^* \).

(II) The principal can not observe the effect level of the agent.

Derivation of calculus to Equation (14) with respect to ‘\( a \)’ and as such we can get:
\[
\frac{\partial E(U)}{\partial a} = \beta - \gamma a = 0 \quad a = \frac{\beta}{\gamma}
\]

This is not a typical Pareto optimal state, because if \( \beta = 0 \), then \( a = 0 \). It means that agent’s income is independent with output. The agent will choose \( a = 0 \), but he has fixed income \( a \).

The optimized mathematics model is therefore given as:
\[
\text{Max} - \alpha + (1 - \beta)a
\]
\[\alpha, \beta, \alpha \]

(II) s. t. \( \alpha + \beta a - \frac{1}{2}\gamma a^2 - \frac{1}{2}\rho\beta^2\sigma^2 \geq U^0 \) \hspace{1cm} (24)

(II) \( a = \frac{\beta}{\gamma} \) \hspace{1cm} (25)

The problem of the principal is how to determine \( \beta \) and \( \beta^* \) to achieve the maximization.

Substitute the value of \( \alpha \) in Equation (24) and the value of \( a \) in Equation (25) into Equation (23) yields:
\[
\text{Max} \left( \frac{\beta}{\gamma} - \frac{1}{2}\rho\beta^2\sigma^2 - \frac{1}{2}\gamma(\beta^2 - U^0) \right)
\]

Derivation calculus to Equation (26) with respect to \( \beta \) sets the derivation to zero and as such we can get:
\[
\frac{1}{\gamma} - \rho\beta^2 - \frac{\beta}{\gamma} = 0 \quad \beta^* = \frac{1}{1 + \gamma\rho\sigma^2}
\]

From the aforementioned statement, we have the following results:

1. \( \beta^* > 0 \), agent takes some risks;
2. \( \beta^* \) is a function of \( \gamma, \rho, \) and \( \sigma^2 \). It is a decreasing function. Since \( c(a) = \frac{1}{2}\gamma a^2 \) if \( \gamma \) becomes larger, \( c(a) \) becomes large and the principal's cost function becomes large. \( \beta^* \) is smaller, the agent bears smaller risk.
3. Since \( \beta = \gamma a \), if ‘\( a \)' is under the same circumstances, \( \gamma \) becomes larger, \( \beta \) becomes large.

Cost-saving efforts are given as:
\[
\frac{1}{2}\rho\beta^2\sigma^2 = \frac{\rho\sigma^2}{2(1 + \gamma\rho\sigma^2)} > 0
\]

The principal can observe the effect level of the agent.

From Equation (20), \( a^* = \frac{1}{\gamma} \) and the principal can not observe the effect level of the agent. From Equation (25), \( a = \frac{\beta}{\gamma} \). Therefore, expected net loss of output becomes:
\[
\Delta a = a^* - a = \frac{1}{\gamma} - \frac{1}{\gamma(1 + \gamma\rho\sigma^2)} = \frac{\rho\sigma^2}{1 + \gamma\rho\sigma^2} > 0
\]

Cost-saving efforts are given as:
\[
\Delta c = c(a^*) - c(a) = \frac{(2 + \gamma\rho\sigma^2)\rho\sigma^2}{2(1 + \gamma\rho\sigma^2)^2}
\]
\[ \Delta a - \Delta c = \frac{\gamma(\rho \sigma^2)^2}{2(1 + \gamma \rho \sigma^2)^2} > 0 \quad (31) \]

Agent’s total cost is given as:
\[ AC = \Delta a + \Delta c = \frac{\rho \sigma^2}{1 + \gamma \rho \sigma^2} + \frac{(2 + \gamma \rho \sigma^2) \rho \sigma^2}{2(1 + \gamma \rho \sigma^2)^2} > 0 \quad (32) \]

**ILLUSTRATION**

Once a project principal intends to sign a principal-agent contract with the agent, then, the cost factor is \( \gamma = 0.0001 \); the absolute measure of risk aversion \( \rho = 0.1 \); the supervisor’s retained income level \( U^0 = 2000 \); \( \theta \sim N(0, 300^2) \). The following illustrations are made:

1. **The principal can observe the effect level of the agent.** According to Equations (20), (21), we get:
   \[ a^* = \frac{1}{\gamma} = 10000 \]
   \[ \alpha^* = U^0 + \frac{1}{2\gamma} = 2000 + 5000 = 7000 \]

2. **The principal can not observe the effect level of the agent.**

   According to Equations (24), (25), (27), we get:
   \[ \beta^* = \frac{1}{1 + \gamma \rho \sigma^2} = \frac{1}{1 + 0.0001 \times 0.1 \times 300^2} = 0.5266 \]
   \[ a = \frac{\beta^*}{\gamma} = \frac{0.5266}{0.0001} = 5266 \]
   \[ \alpha = -\beta a + \frac{1}{2} \rho \sigma^2 + \frac{1}{2} \rho^2 \sigma^2 \cdot U^0 = -0.5266 \times 5266 + \frac{1}{2} \times 0.0001 \times 5266^2 + \frac{1}{2} \times 0.1 \times 0.5266^2 \times 90000 + 2000 = 1861.346 \]
   \[ S(\pi) = \alpha + \beta a = 1861.346 + 0.5266 \times 5266 = 4634.422 \]

The expected net loss of output is given as:
\[ \Delta a = a^* - a = \frac{\rho \sigma^2}{1 + \gamma \rho \sigma^2} = 4734 \]

The cost-saving efforts are:
\[ \Delta c = c(a^*) - c(a) = \frac{(2 + \gamma \rho \sigma^2) \rho \sigma^2}{2(1 + \gamma \rho \sigma^2)^2} = 1386.538 \]

The Incentive costs:
\[ \Delta a - \Delta c = \frac{\gamma(\rho \sigma^2)^2}{2(1 + \gamma \rho \sigma^2)^2} = 1386.518 \]

The Agent’s total cost is given as:
\[ AC = \Delta a + \Delta c = \frac{\rho \sigma^2}{2(1 + \gamma \rho \sigma^2)^2} = 4284 \]

**CONCLUSION**

In this study, under asymmetric information, we found that, moral hazard and adverse selection mechanism of inhibition is designed to achieve reasonable contract supervision. Enterprises provide incentives, the manager’s level of effort and risk-averse attitude have absolute relationship. Managers more risk aversion, enterprise has the smaller incentive. The stronger the managers, professional and higher level of effort, the greater the degree of incentive companies. From the perspective of agency cost, high-level managers within the organization should have to assume more responsibility for enterprise risk than grass-roots workers. Through an illustration, we successfully validated the model.

**RECOMMENDATION**

The recommendations of this study are:

1. **Design management system main objectives in reducing conflict as principal and agent, but can not completely eliminate the agency problem. In information asymmetry, the agent (managers) is based on self-interest motive arising from speculation.**
2. **In order to reduce the risk, enterprise is designed of punishment mechanism, but to promote the agent (manager) encroach more enterprise resources, such as punishment reflection on enterprise lost profits.**
3. **The development of an effective incentive contract objectives and modalities provides information on the company and should also consider shared responsibility,**
professional competence and an additional bonus payment.

REFERENCES


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