A new proposal for black holes

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ABSTRACT

The understanding of what a black hole is like is not easy and may not yet be well understood. The introduction of canonical quantization into the issue has not been significant to our understanding. However, introducing affine quantization, a new procedure, offers a very unusual expression that seems to be plausible, and quite profound as well.

Key words: Black hole, affine quantization, model.

INTRODUCTION

We offer three models of the features of black holes. Two of those models are plausible, but one of them is deliberately false to make a point. The first model resembles our present view of black holes. The second model, which is only a mathematical dream, offers an impossible approach to a different result. The third model, introducing an unusual quantization feature, shows that the second model can actually become possible, thanks to affine quantization, a procedure that is briefly defined.

MODEL 1: A ROUGH VERSION OF OUR PRESENT UNDERSTANDING

The model classical Hamiltonian, for a single particle in this first case, is \( H_1 = \frac{\vec{p}^2}{2} + g(\vec{q}^2)^r \) with \( g \) is the coupling constant and \( r \) is the power of the interaction term. This potential term simply represents any, and all, potentials that would continuously point strongly toward \( \vec{q}^2 = 0 \). This model is then copied hugely so that, in reality, normal friction leads to a pileup at the region around \( \vec{q}^2 = 0 \). If that pileup is strong enough, it can burst a hole in the space around \( \vec{q}^2 = 0 \). Having done so, it follows that some trash can fall into the hole. Indeed, it is even suggested that the hole may be connected to a different hole from which some trash from the first hole comes out of the second hole, now in a different space.

Using canonical quantization, which leads to our primitive quantum Hamiltonian, \( \mathcal{H}_1 = \frac{\vec{p}^2}{2} + g(\vec{q}^2)^r \) does not significantly modify the outlook of the classical Hamiltonian.

MODEL 2: A VERY DIFFERENT TYPE OF POTENTIAL IS INTRODUCED

In the present case, our classical Hamiltonian becomes

\[
H_2 = \frac{\vec{p}^2}{2} + \zeta^2(2\vec{Q}^2 + b^2)/(\vec{Q}^2 - b^2)^2 + g(\vec{Q}^2)^r
\]

(1)

in which \( \vec{Q}^2 > b^2 \) while \( \vec{Q}^2 = b^2 \) represents any point on the edge of the hole. Following the rules of canonical quantization, the quantum Hamiltonian for this example is

\[
\mathcal{H}_2 = \frac{\vec{\mathcal{P}}^2}{2} + \zeta^2(2\mathcal{Q}^2 + b^2)/(\mathcal{Q}^2 - b^2)^2 + g(\mathcal{Q}^2)^r
\]

(2)

in which \( \mathcal{Q}^2 > b^2 \).

Evidently, this extra potential term prevents any
elements from falling into the hole no matter how much trash piles up.
This example is chosen only to imagine what could happen if it had been possible.

**MODEL 3: WHAT AFFINE QUANTIZATION CAN DO**

**An example like Model-2 that is real**

The third model will be given now to compare with the former models. The following section will prove that nature may have chosen the third expression.

Although, while the classical Hamiltonian is again simple, that is, \( H_3 = \frac{p^2}{2} + g(\bar{q}^2)^{r} \), along with \( \frac{\bar{q}^2}{b^2} > \frac{b^2}{r} \), we find that the affine quantization for this model is given by

\[
H_3 = \frac{p^2}{2} + \hbar^2 (2\bar{q}^3 + b^3)/\left(\bar{q}^2 - b^3\right)^2 + g(\bar{q}^3)^r
\]

(3)

**A brief introduction to affine quantization**

**The basic story**

Initially, we focus on a single coordinate space where \( 0 < q < \infty \), while \( p \) has its usual values. But now \( p^4 \neq p \). To overcome this, we use the dilation variable and the coordinate, specifically, \( d = pq \) & \( q > 0 \), along with \( D = \left[ p^4Q + QP \right]/2 = (P^4) & Q (Q > 0) \), as the new basic classical and quantum variables. The kinetic term, \( p^2 = d^2/q^2 \Rightarrow D(Q^{-2})D = P^2 + (3/4)h^2/Q^2 \) (Klauder, 2020). The proof of this analysis is correct and follows from the standard harmonic oscillator, with \( H = (p^2 + q^2)/2 \Rightarrow H = (P^2 + Q^2)/2 \), and the eigenvalues for this model are \( E_n = \hbar(n + 1/2) \) for \( n = 0, 1, 2, \ldots \). Next, the half-harmonic oscillator, in which \( q > 0 \), leads to \( H_3 = (p^2 + q^2)/2 = (d^2/q^2 + q^2)/2 \)

\[
[D(Q^{-2})D + Q^2]/2 = [P^2 + (3/4)h^2/Q^2 + Q^2]/2,
\]

which has the new eigenvalues (Gouba, 2005) \( E_{n>0} = 2\hbar(n+1) \) for the same \( n \)-values **Both of these eigenvalues are equally spaced, and both of them employ “2” which is “natural” for the full- and half-harmonic oscillators**! The usual \( h \)-term seems to be unexpected and even remarkably strong near \( q = 0 \), but there it is! Note that the \( \hbar \)-term leads to an infinite, physically relevant, wall that is required by the quantum physics of an affine quantization. This quantum potential ensures that space is present only where \( q > 0 \), and space is absent where \( q < 0 \). **Affine quantization has created a ‘quantum wall’ all by itself!** Canonical quantization cannot do that.

**Removing two points of coordinate space**

In this section we focus on a single line in which we remove \( q = \pm b \), with \( 0 < b < \infty \). We skip the center part \( |q| < b \), and keep only the outer space regions where \( |q| > b \). In this case, the classical Hamiltonian is simply \( H_{\bar{q} > b} = p^2/2 + V(q) \), while the quantum Hamiltonian using affine quantization (Fantoni and Klauder, 2012) leads to

\[
H_{\bar{q} > b} = \frac{1}{2}\left[ p^2 + \hbar^2(2\bar{q}^3 + b^3)/(\bar{q}^2 - b^3)^2 + V(Q) \right],
\]

(4)

with a potential \( V(Q) \) that pushes particles toward the center.

We now take a big step to enhance our simple model into a related field theory model.

**Promotion of a simple model to a field theory**

This example can be extended to a field formulation, and using Schrödinger’s representation, we would find, after a considerable rescaling of the operators (Fantoni and Klauder, 2022), it leads to the quantum Hamiltonian,

\[
H_{\varphi(x) > b} = \int \left\{ \frac{1}{2} \left[ \varphi'(x)^2 + \hbar^2(2\varphi(x)^3 + b^3)/(\varphi(x)^2 - b^3)^2 \right] + m^2 \varphi(x)^2 + g(\varphi(x)^2)^r \right\} dx.
\]

(5)

**Two and three spatial dimensions, choosing symmetric for simplicity**

The previous dealt with a single spatial dimension. In the remaining sections, \( x \) will stand for \( x = (x_1, x_2) \) or \( x = (x_1, x_2, x_3) \). The vector arrows on top of several terms signal 2 or 3 dimensions. By adding up independent elements, we find the vector version, again with Schrödinger’s representation, is given by

\[
H_{\vec{\varphi}(x) > b} = \int \left\{ \frac{1}{2} \left[ \vec{\varphi}'(x)^2 + \hbar^2[2\vec{\varphi}(x)^3 + b^3]/[(\vec{\varphi}(x)^2 - b^3)^2] \right] + m^2 \vec{\varphi}(x)^2 + g(\vec{\varphi}(x)^2)^r \right\} dx.
\]

(6)

**NON-FIXED SPATIAL AND TIME VARIATIONS**

While for clarity, having used a fully symmetric formulation, it is easy to change that.
All that is necessary is to let \( b b(x, t) \), which would introduce flexible changes in both space and time. Beyond that, trash may pile up in “equator-like” regions leaving polar-like regions essentially free of trash, and therefore making the black hole ‘visible’.

**SUMMARY**

In this study, we have shown a possible view of black holes, including an affine quantization, which involves a reduction of space for it to work well. Focusing now on \( x = (x_1, x_2, x_3) \), our view of black holes, at least mathematically, ‘resemble’ the Big Bang, which in that case is where space essentially expands, or, much later, perhaps, deaces. In fact, much of our analysis could just be ‘turned around’, where now \( \phi^2 < B^2 \), rather then our previous \( \phi^2 > b^2 \) to exhibit some features like the Big Bang itself. Let’s see what that might look like.

For example, using the Schrödinger representation, we offer a quantum Hamiltonian that appears relevant to the Big Bang (BB), namely

\[
H_{BB}(t) = \int \{ \overline{\partial}^2(x,t)^2 + \hbar^2 (2 \overline{\partial}^2(x,t)^2 + B(x,t)^2)/(B(x,t)^2 - \overline{\partial}^2(x,t)^2) \} \, dx,
\]

along with \( \overline{\partial}^2(x,t)^2 < B(x,t)^2 \). After the Big Bang occurred, a huge pressure to expand space outwards arose. This would force \( B(x, t) \) to have a huge, outwards, velocity, that is, \( B(x, t) \) was positive and huge. Afterwards, it may have only slowed a little while adding more space. As the quantum wall moved outward, the new space came along with it. Perhaps, that happened in the past, or will again, contract, in which case \( B(x, t) \) would now retract. Perhaps, space could be elastic, but only moves relatively slowly.

This view could be an estimate of how our Universe evolved, while also creating black holes along the way. The presence or absence of space forbid the trash from entering space-free regions. That is due to properties of the affine quantization procedures.

**REFERENCES**


**APPENDIX**

Uses of AQ for solving or improving results of several quantum field theories, have shown that AQ can be very useful to those problems. To illustrate that fact, several of published articles have been added to this paper. They have shown that AQ has improved our general knowledge, for which the authors are thankful; see (Fantoni, 2021; Fantoni and Klauder, 2021, 2021, 2022).