Fractal interpolation surface under contraction IFS

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ABSTRACT

In this paper, the problem of fractal interpolation surface as the fixed point of certain Read-Bajraktarevic operators was introduced. It was shown that the graph of fractal interpolation surface is the attractor of underlying contractive Iterated Function Systems (IFS) by exhibiting a generalized metric $d_f$.

Key words: Contractive IFS, fractal interpolation, fractal interpolation surface.

INTRODUCTION

Fractal interpolation is a new important technique proposed by Barnsley in (1986) and applied in many researches such as computer graphics, materials and seismology etc (Barnsley, 2000; Barnsley and Vince, 2010; Peng and Wang, 2011; Feng et al., 2012). It offers an alternative to traditional interpolation methods, aiming primarily at the data which present detail at different scales or some degree of self-similarity. The intrinsic characteristics in natural objects implies that an irregular and non-smooth structure is inconveniently captured by elementary functions such as polynomials (Drakopoulos and Manousopoulos, 2012).

Standard framework for describing and analyzing self-referential sets used as IFS is deterministic fractals (Barnsley, 2000; Barnsley and Vince, 2010). IFS with contraction function is called a contractive IFS (Barnsley and Vince, 2013; Hutchinson, 1981). In addition, the attractors of IFSs have many applications including image compression (Yuval, 1994). Furthermore, a contractive IFS on a complete metric space is proven to have a unique attractor. In mathematics, the interpolation technique is based on a function $f(x, y, z)$ which passes through prescribed dataset points. As textures revealed, generally, a non-smooth or even fractal characteristic, a description in terms of fractal geometric method seems reasonable.

In this case, the classical approximation method is replaced by a fractal interpolation procedure (Barnsley and Massopust, 2015). In this situation, we extended and corrected some identified results concerning fractal interpolation functions that are fixed points called Read-Bajraktarevic operators (Massopust, 1994). The attractor of IFS obtained from Theorem 5.2 relates to the fixed point as in Theorem 3.2. This paper introduced fractal interpolation surface and shows the graph is an attractor of an underlying contractive IFS.

FRACTAL INTERPOLATION SURFACE (FIS) PROBLEM

Definition 2.1

Let $I = [a, b]$ and $J = [c, d]$. For arbitrary function $f : I \times J \rightarrow \mathbb{R}$, the graph of $f$ denoted by $G(f) = \{(x, y, f(x, y)) : (x, y) \in D\}$ is the image of function $f$ in area $D$. (Without ambiguity will be denoted by $G(f)$ for short). We denote $C[D]$ as the set of all continuous functions in area $D$.

Definition 2.2

Given scattered data sets:

$$\{(x_i, y_j, z_{ij}) \in \mathbb{R}^3, i = 0, 1, 2, ..., N, j = 0, 1, 2, ..., M\}$$ and
The interpolation surface problem is to find a continuous function \( f(x, y) \in C[D] \) satisfying:

\[
f(x_i, y_j) = z_{ij}, \quad (i = 0, 1, 2, \ldots, N, j = 0, 1, 2, \ldots, M).
\]

Let \( \{(x_i, y_j, z_{ij})\}_{i=0,j=0}^{N,M} \) be the set of interpolation data, and we call \( \{(x_i, y_j)\}_{i=0,j=0}^{N,M} \) the set of interpolation nodes and \( \{ (z_{ij})\}_{i=0,j=0}^{N,M} \) the set of interpolation shaped values. \( D = I \times J = [a,b] \times [c,d] \) is interpolation area and \( K = D \times I \) is interpolation space.

**FRACTAL INTERPOLATION SURFACE AS FIXED POINTS OF OPERATORS**

Let \( \Delta = \{(x_i, y_j, z_{ij})\} \in [i, 1, 2, \ldots, N, j = 1, 2, \ldots, M} \) denote the Cartesian co-ordinates of a finite set of points in the Euclidean space with \( x_0 < x_1 < \ldots < x_N \), \( y_0 < y_1 < \ldots < y_M \). We can then take:

\[
x_0 = y_0 = a < \min\{x_i, y_j\} \quad \text{and} \quad x_{N+1} = y_{M+1} = b > \max\{x_i, y_j\}.
\]

Let \( I = [x_0, x_{N+1}], \quad I_N = [x_N, x_{N+1}] \) be closed intervals and \( I_k = [x_k, x_{k+1}], \quad k = 0, 1, \ldots, N-1 \), and let \( J = [y_0, y_{M+1}], \quad J_M = [y_M, y_{M+1}] \) be closed intervals and \( J_k = [y_k, y_{k+1}], \quad k = 0, 1, \ldots, M-1 \).

For \( k = 0, 1, \ldots, N \), let \( l_k : I \to I_k \) be a continuous bijection defined by:

\[
l_k(x) := x_k + \frac{x_{k+1} - x_k}{x_{N+1} - x_0}(x - x_0).
\]

Then, \( l_k \) is a continuous bijection from \( [x_0, x_{N+1}] = [a,b] \) to \( I_k, k = 0, 1, \ldots, N-1 \), and \( l_N \) is a continuous bijection from \( [x_0, x_{N+1}] = [a,b] \) to \( I_N \). Similarity for \( k = 0, 1, \ldots, M \), let \( m_k : J \to J_k \) be a continuous bijection defined by:

\[
m_k(y) := y_k + \frac{y_{k+1} - y_k}{y_{M+1} - y_0}(y - y_0).
\]

(2)

From Equations (1) and (2) it follows that there exists a \( \lambda \in (0,1) \) such that:

\[
\| (l_1(s_1), \tilde{l}_J(t_1)) - (l_1(s_2), \tilde{l}_J(t_2)) \| \leq \lambda \| (s_1, t_1) - (s_2, t_2) \|
\]

(3)

for all \( (s_1, t_1), (s_2, t_2) \in [a,b]^2 \), \( \forall 0 \leq i \leq N, 0 \leq j \leq M \).

Note that \( [a,b] = U_k^{N} I_k = U_k^{M} J_k \) with disjoint union. This gives a map \( L : [a,b]^2 \to [a,b]^2 \) be such that:

\[
L(x, y) = (l^{-1}_1(x), \tilde{l}_J^{-1}(y)) \text{ for } (x, y) \in I \times J, 0 \leq i \leq N, 0 \leq j \leq M.
\]

Then, the set of discontinuous points of \( L(x, y) \) is \( (U_k^{N} \{ x_i \} \times [a,b]) \cup (U_k^{M} [a,b] \times \{ y_j \}). \) Recall that:

\[
C = C([a,b]^2) \quad \text{is the set of continuous functions from} \quad [a,b]^2 \quad \text{to} \quad I \quad \text{and} \quad G(f) \text{ is the graph of} \ f \in C, \text{that is:}
\]

\[
G(f) = \{(x, y, f(x, y)) : (x, y) \in [a,b]^2\}.
\]

It is well-known that \( (C, d_\infty) \) is a complete metric space where:

\[
d_{\infty}(f, g) = \max_{(x,y)\in[a,b]^2} |f(x, y) - g(x, y)|, \quad f, g \in C.
\]

Let:

\[
C_1 = \{ f \in C : f(a,b) = 0, f(a,y) = f(b,y) \text{ and } f(x,a) = f(x,b), \forall x, y \in [a,b] \},
\]

And:

\[
C_2 = \{ f \in C_1 : f(x_i, y_j) = z_{ij}, i = 1, 2, \ldots, N, j = 1, 2, \ldots, M \}.
\]

Note that \( C_1 \) and \( C_2 \) are closed sub-spaces of \( C \) with \( C_2 \subset C_1 \subset C \). The functions in \( C_2 \) interpolates the data set \( \Delta = \{(x_i, y_j, z_{ij})\} \). Let \( S : [a,b]^2 \to I \) be a bounded function with possible discontinuous points lying in:

\[
\tilde{l}_k(y) := y_k + \frac{y_{k+1} - y_k}{y_{M+1} - y_0}(y - y_0).
\]
\{(x_i, y_j) : 1 \leq i \leq N, 1 \leq j \leq M\} and satisfying that:

\[S(a, y) = S(b, y) \text{ and } S(x, a) = S(x, b), \forall x, y \in [a, b].\]

Let \(s = \sup\{\|S(x, y)\|: (x, y) \in [a, b]^2\}\). Let \(u(x, y) \in C_1\) an \(h(x, y) \in C_2\). Define Read-Bajraktarevic operator \(T : C_1 \to C_2\) by:

\[Tg(x, y) := h(x, y) + S(x, y)(g(L(x, y)) - u(L(x, y))).\] (4)

\(T\) is a form of Read-Bajraktarevic operator as defined by Massopust (1994).

**Theorem 3.1**

Let \(u \in C_1\) and \(h \in C_2\). Let \(g \in C_1\) and \(T\) be defined as in Equation (4), then \(Tg \in C_2\). That is, \(T\) is an operator from \(C_1\) into \(C_2\).

**Proof:** Firstly, we will prove that \(Tg(x, y)\) is continuous.

To this end fixed \(g_1 \in C_1, \forall \varepsilon > 0, \exists \delta = \frac{1}{s}\), for all \(g_2 \in C_1,\) such that \(d_s(g_2(x, y), g_1(x, y)) < \delta\), then \(d_s(Tg_2(x, y), Tg_1(x, y)) < \varepsilon\). From Equation (4) we have:

\[d_s(Tg_2(x, y), Tg_1(x, y)) = \max_{(x, y) \in [a, b]^2} |f_{g_2(x, y)} - f_{g_1(x, y)}|\]

\[= \max_{(x, y) \in [a, b]^2} |S(x, y)(g_2(L(x, y)) - g_1(L(x, y)))|\]

\[\leq s \max_{(x, y) \in [a, b]^2} |g_2(1^{-1}_i(x), 1^{-1}_j(y)) - g_1(1^{-1}_i(x), 1^{-1}_j(y))|\]

\[\leq s \delta = \varepsilon.

Where \(1_i^{-1}(x) = x_i\) and \(1_j^{-1}(y) = y_j\). Secondly, we can check that \(Tg(x, y)\) satisfies the \(C_1\) and \(C_2\) conditions given as:

\[Tg(a, a) = h(a, a) + S(a, a)(g(L(a, a)) - u(L(a, a)))
\]

\[= h(a, a) + S(a, a)(g(1_i^{-1}(a), 1_j^{-1}(a)) - u(1_i^{-1}(a), 1_j^{-1}(a)))
\]

\[= h(a, a) + S(a, a)(g(a, a) - u(a, a)) = 0.

Note that \(1_i^{-1}(a) = a\) and \(1_j^{-1}(a) = a\) for \(i = j = 0\). It is easy to verify that \(Tg(a, y) = Tg(b, y)\) from our knowledge \(g, u \in C_1\) that is, \(g(a, y) = g(b, y)\) and \(u(a, y) = u(b, y)\), \(h \in C_2\), \(C_2 \subset C_1\) (that is, \(h(a, y) = h(b, y)\)) and \(S\) satisfying that \(S(a, y) = S(b, y)\). Then we have:

\[Tg(a, y) = h(a, y) + S(a, y)(g(L(a, y)) - u(L(a, y)))
\]

\[= h(a, y) + S(a, y)(g(1_i^{-1}(a), 1_j^{-1}(y))) - u(1_i^{-1}(a), 1_j^{-1}(y)))
\]

\[= h(a, y) + S(a, y)(g(a, y) - u(a, y)))
\]

\[= h(b, y) + S(b, y)(g(b, y) - u(b, y)))
\]

\[= Tg(b, y).

Similarly, it is easy to verify that \(Tg(x, a) = Tg(x, b)\).

Finally:

\[Tg(x, y) = h(x, y) + S(x, y)(g(L(x, y)) - u(L(x, y)))
\]

\[= h(x, y) + S(x, y)(g(1_i^{-1}(x), 1_j^{-1}(y))) - u(1_i^{-1}(x), 1_j^{-1}(y)))
\]

\[= h(x, y) + S(x, y)(g(a, a) - u(a, a))
\]

\[= h(x, y) = z_0,

Where \(1_i^{-1}(x) = x_0 = a\) and \(1_j^{-1}(y) = y_0 = a\) for \(i = 0, 1, ..., N, j = 0, 1, ..., M\), that completes the proof W.

**Theorem 3.2**

The operator \(T : C_1 \to C_2\) satisfies:

\[d_s(Tf_1(x, y), Tf_2(x, y)) \leq s d_s(f_1(x, y), f_2(x, y))\]

for any \(f_1, f_2 \in C_1\).

Thus, \(T\) is contractive and possesses a unique fixed point \(f \in C_2\) when \(s \in [0, 1]\).

**Proof:** Observe that:

\[d_s(Tf_1(x, y), Tf_2(x, y)) = \max_{(x, y) \in [a, b]^2} |[f_{Tf_1(x, y)} - f_{Tf_2(x, y)}]|\]

\[= \max_{(x, y) \in [a, b]^2} |S(x, y)f_1(L(x, y)) - f_2(L(x, y))|\]

\[\leq s d_s(f_1(x, y), f_2(x, y)).

When \(s \in [0, 1]\), \(T\) has a unique fixed point \(f \in C_1\) by contraction mapping theorem. Since \(C_2 \subset C_1 \subset C\) and
\((C_2,d_{\omega})\) is closed subspaces of \((C,d_{\omega})\), hence, it is complete and follows that \(f = fT \in C_2, W\)

The fixed point \(f \in C_2\), interpolates the data \(\Delta = \{(x_i, y_j, z_{ij}) \in i^3, i = 1, 2, ..., N, j = 1, 2, ..., M\}\), as an example of a fractal interpolation function (Barnsley, 1986; Edgar, 2008) and satisfies \(f = \lim_{k \to \infty} T^k(f_0)\) for any \(f_0 \in C_1\). The proof of the contraction mapping theorem also gives an estimate for the rate of convergence:

\[
\left\| f - T^k(f_0) \right\|_\infty \leq \frac{s^k}{1-s} \left\| f_0 - f_0 \right\|_\infty.
\]

In addition, an estimate for the operator \(T\) is:

\[
\left\| T \right\|_\infty = \sup\left\{ \left\| T f_0 \right\|_\infty : f_0 \in C_1, \left\| f_0 \right\|_\infty = 1 \right\}.
\]

THE METRIC SPACE \([(a,b)^2 \times \bar{i}, d_q)\]

Fix \(\alpha > 0, \beta > 0\) and \(q : [a,b]^2 \to \bar{i}\). For \((s_1, s_2, s_3), (t_1, t_2, t_3) \in [a,b]^2 \times \bar{i}\) we have the following metric:

\[
d_q((s_1, s_2, s_3), (t_1, t_2, t_3)) = \alpha \left| s_1 - s_2 \right| + \beta \left| s_1 - q(s_1, s_2) \right| + \beta \left| t_1 - q(t_1, t_2) \right|.
\] (5)

Proposition 4.1

The \(d_q\) defined by Equation (5) is a metric on \([a,b]^2 \times \bar{i}\).

If \(q\) is continuous, then \([a,b]^2 \times \bar{i}, d_q)\) is a complete metric space.

Proof: Clearly \(d_q((s_1, s_2, s_3), (t_1, t_2, t_3)) = d_q((t_1, t_2, t_3), (s_1, s_2, s_3)) \geq 0\).

Let \(d_q((s_1, s_2, s_3), (t_1, t_2, t_3)) = 0\). Then, \(\alpha \left| s_1 - s_2 \right| + \beta \left| s_1 - q(s_1, s_2) \right| + \beta \left| t_1 - q(t_1, t_2) \right| = 0\) which implies \(s_1 = t_1, s_2 = t_2\) and then \(s_3 = t_3\). Finally, for \((s_1, s_2, s_3), (t_1, t_2, t_3), (r_1, r_2, r_3) \in i^3\) we have:

\[
d_q((s_1, s_2, s_3), (t_1, t_2, t_3)) + d_q((t_1, t_2, t_3), (r_1, r_2, r_3)) + d_q((r_1, r_2, r_3), (s_1, s_2, s_3)) \geq 0.
\]

This gives triangle inequality. Suppose that \(q\) is continuous and \(\{(s_k, t_k, r_k)\}_{k=1}^{\infty}\) is a Cauchy sequence in \([a,b]^2 \times \bar{i}, d_q)\). Thus, for any given \(\varepsilon > 0\) we can find an integer \(N(\varepsilon)\) such that:

\[
d_q((s_k, t_k, r_k), (s_l, t_l, r_l)) < \varepsilon \text{ for all } k, l > N(\varepsilon).
\]

It follows that \(\{(s_k, t_k)\}_{k=1}^{\infty}\) is a Cauchy sequence in \([a,b]^2\) with respect to Euclidean norm and that \(\{r_k - q(s_k, t_k)\}_{k=1}^{\infty}\) is a Cauchy sequence in \(i\) with respect to Euclidean norm. Thus \(\lim_{k \to \infty} (s_k, t_k) = (s', t') \in [a,b]^2\) and

\[
\lim_{k \to \infty} (r_k - q(s_k, t_k)) = \eta \in i.\text{ The latter implies that }\lim_{k \to \infty} r_k = \eta + q(s', t')\text{ by the continuity of }q.\text{ Finally, we have:}
\]

\[
\lim_{k \to \infty} d_q((s_k, t_k, r_k), (s', t', \eta + q(s', t'))) = \lim_{k \to \infty} \alpha \left| s_k - t_k \right| + \beta \left| s_k - q(s_k, t_k) \right| + \beta \left| t_k - q(t_k, t_2) \right| = 0.
\]

This implies that \((s_k, t_k, r_k)\) converges to \((s', t', \eta + q(s', t'))\) and as such \(([a,b]^2 \times \bar{i}, d_q)\) is a complete metric space \(W\).

FRACTAL INTERPOLATION SURFACE AS ATTRACTORS OF ITERATED FUNCTION SYSTEMS

Recall from definition of Read-Bajraktarevic operator:

\(Tg(x, y) := h(x, y) + S(x, y)(g(L(x, y)) - u(L(x, y)))\).

For the fixed point \(f\) of \(T\) in Theorem 3.2, let \(\zeta > 0\), then define:

\[
X_\zeta = \{(x, y, z) \in [a,b]^2 \times \bar{i} : |z - f(x, y) \leq \zeta\}.
\] (6)
For \( 0 \leq i \leq N, 0 \leq j \leq M \) we define \( w_{ij} : [a, b]^2 \times i \rightarrow [a, b]^2 \times i \) by:

\[
w_{ij}(x, y, z) = (l_i(x), \tilde{l}_j(y), h(l_i(x), \tilde{l}_j(y))) + S(l_i(x), \tilde{l}_j(y))(z - u(x, y)),
\]

(7)

Where the functions are the same as those in Equation (4) with \( s \in [0, 1) \). Then we have the following properties:

1. \( w_{ij}(a, a, 0) = (l_i(a), \tilde{l}_j(a), h(l_i(a), \tilde{l}_j(a))) + S(l_i(a), \tilde{l}_j(a))(0 - u(a, a))) = (x_i, y_j, z_{0i,j}). \)

2. \( w_{ij}(b, b, 0) = (l_i(b), \tilde{l}_j(b), h(l_i(b), \tilde{l}_j(b))) + S(l_i(b), \tilde{l}_j(b))(0 - u(b, b))) = (x_i, y_j, z_{0i,j}). \)

3. \( w_{ij}(b, b, 0) = (l_i(b), \tilde{l}_j(b), h(l_i(b), \tilde{l}_j(b))) + S(l_i(b), \tilde{l}_j(b))(0 - u(b, b))) = (x_i, y_j, z_{0i,j}). \)

Now for \( B \subseteq [a, b]^2 \times i \) let:

\[
W(B) = \bigcup_{0 \leq i \leq N, 0 \leq j \leq M} w_{ij}(B).
\]

(8)

**Lemma 5.1**

Let \( f \) be the fixed point of \( T \) as in Theorem 3.2. Let \( S \) be defined with \( s \in [0, 1) \). Let \( X_{\varepsilon} \) and \( W \) be defined by Equations (6) and (8), respectively. Then, \( W(X_{\varepsilon}) \subseteq X_{\varepsilon} \).

**Proof:** By considering \( X_{\varepsilon} \) given in Equation (6), we verify that for any \( (x, y, z) \in X_{\varepsilon} \) and any \( 0 \leq i \leq N, 0 \leq j \leq M \) we have \( w_{ij}(x, y, z) \in X_{\varepsilon} \). By Equation (7) we have:

\[
w_{ij}(x, y, z) = (l_i(x), \tilde{l}_j(y), h(l_i(x), \tilde{l}_j(y))) + S(l_i(x), \tilde{l}_j(y))(z - u(x, y)).
\]

Recall that \( f \) be the unique fixed point of \( T \) as in Theorem 3.2. Then:

\[
Tf(x, y) = f(l_1(x), \tilde{l}_1(y)) = h(l_1(x), \tilde{l}_1(y)) + S(l_1(x), \tilde{l}_1(y))(f(L_1(x), \tilde{l}_1(y)) - u(L_1(x), \tilde{l}_1(y))).
\]

First, we like to point out an important fact that for any \( v \in C_1 \), for any \( (x, y) \in [a, b]^2 \)

\[
v(L(l_i(x), \tilde{l}_j(y))) = v(x, y) \quad \text{for} \quad 0 \leq i \leq N, 0 \leq j \leq M.
\]

(9)

Thus, by Equation (4) we have:

\[
\| h(l_i(x), \tilde{l}_j(y)) + S(l_i(x), \tilde{l}_j(y))(z - u(x, y)) - f(l_i(x), \tilde{l}_j(y)) \|
= \| S(l_i(x), \tilde{l}_j(y))(z - u(x, y)) - S(l_i(x), \tilde{l}_j(y))(f(L(l_i(x), \tilde{l}_j(y))) - u(L(l_i(x), \tilde{l}_j(y))) \|
= \| S(l_i(x), \tilde{l}_j(y))(z - u(x, y)) - S(l_i(x), \tilde{l}_j(y))(f(x, y) - u(x, y)) \|
= \| S(l_i(x), \tilde{l}_j(y))(z - f(x, y)) \| < \varepsilon.
\]

**Theorem 5.2**

Let \( f \in C_2 \) be the fixed point of \( T \) as in Theorem 3.2. Let \( s \) be defined with \( s \in [0, 1) \). Let \( l_\lambda \in (0, 1) \) be given in Equation (3). Suppose that \( S : [a, b]^2 \rightarrow [-s, s] \) satisfies that there exists \( \lambda_s \) such that:

\[
|S(s_1, t_1) - S(s_2, t_2)| \leq \lambda_s |(s_1, t_1) - (s_2, t_2)| \quad \text{for all} \quad (s_1, t_1), (s_2, t_2) \in [a, b]^2.
\]

Then the IFS \( W = \{ w_{ij} : i = 0, 1, ..., N, j = 0, 1, ..., M \} \) is contractive on the metric space \((X_{\varepsilon}, d_f)\) with \( \alpha = 1 \) and

\[
0 < \beta < \frac{1 - \lambda_s}{\lambda_s} \in [0, 1) \text{ in Equation (5).}
\]

In particular, the IFS \( W \) have a unique attractor \( G(f) \), with basin of attraction \([a, b]^2 \times i\).

**Proof:** Firstly, from the aforementioned definition of \( w_{ij} \) we have:

\[
w_{ij}(x, y, z) = (l_i(x), \tilde{l}_j(y), h(l_i(x), \tilde{l}_j(y))) + S(l_i(x), \tilde{l}_j(y))(z - f(x, y)).
\]

Take \( h = f \), then we have:

\[
w_{ij}(x, y, z) = (l_i(x), \tilde{l}_j(y), f(l_i(x), \tilde{l}_j(y))) + S(l_i(x), \tilde{l}_j(y))(z - f(x, y)).
\]
Recall that:

\[
d_j((s_1,x_1,i_1,t_1), (x_2,t_2)) = \alpha \|s_1 - x_1\| + \beta \|q(s_1,x_1,i_1) - q(x_2,t_2)\|
\]

Now for any \((s_1,t_1,i_1), (s_2,t_2,i_2) \in X_\zeta\) we have:

\[
d_j(w_i(s_1,t_1,i_1), w_j(s_2,t_2,i_2))
= \alpha \|1(s_1,i_1,t_1) - 1(s_2,i_2,t_2)\|
+ \beta \|f(1(s_1,i_1,t_1)) - f(1(s_2,i_2,t_2))\|
= \alpha \|1(s_1,i_1,t_1) - 1(s_2,i_2,t_2)\|
+ \beta \|f(1(s_1,i_1,t_1)) - f(1(s_2,i_2,t_2))\|
\]

Where \(c = \max\{s, \lambda_1 + \frac{\beta_1 \lambda_2 \zeta}{\alpha}\}\). Since \(\lambda_1 < 1\) we can choose \(\alpha, \beta > 0\) so that \(0 < c < 1\). For example, we can choose \(\alpha=1\) and \(0 < \beta < \frac{1-\lambda_1}{\lambda_1 \lambda_2 \zeta}\). It follows that the IFS \(W\) is contractive on the metric space \((X_\zeta, d_j)\) by Lemma 5.1, and hence, it has a unique attractor \(G(f)\). Since we can choose \(\zeta\) arbitrarily large, it follows that \(W\) has a unique attractor \(G(f)\) with basin of the attractor \([a,b]^2 \times i\).

CONCLUSIONS

A method generating fractal interpolation surfaces on \([a,b]\) domain was presented in this paper. It was proven that the Read-Bajraktarevic operator is contraction and has a unique fixed point that satisfies the \(C_1\) and \(C_2\) conditions. The attractor of IFS is a graph of a continuous function that interpolates a given dataset. This paper has debated that the \([a,b]^2 \times i\) is complete metric space with respect to metric \(d_j\). The results indicated that the IFS is contraction on the metric space and it would be exciting to see that the IFS have a unique attractor with basin of attraction \([a,b]^2 \times i\). The methods offered here can be directly extended to piecewise self-affine fractal interpolation surfaces that are based on Reccurent IFS. We recommend that efforts be geared towards focusing on the extension methods to hidden-variable fractal interpolation surfaces as well as, on the parameters identifications of such surfaces in the future.

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