A mathematical approach to investigate the effect of chip slenderness ratio in primary deformation zone in turning

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ABSTRACT

In extended cutting speed law, Chip Slenderness Ratio (Csr) is a vital parameter and ratio of the depth of cut to feed rate influences machining operations. In this study, the effect of Chip Slenderness Ratio (χ) on shear angle (φ), cutting-edge approach angle (χ), and rake angle (α) was investigated theoretically using Matlab and Minitab 17 software. In conclusion, it was established that φ has an inversely proportional effect on Csr, while χ has a directly proportional effect, but α has an inconsiderable effect on primary deformation zone as a result of affecting moving of removed chip on rake face.

Key words: Chip slenderness ratio, shear angle, cutting edge approach angle, primary deformation zone, orthogonal cutting.

INTRODUCTION

Machining method is widely used as a final stage in manufacturing operations. Machinability depends on many different kinds of parameters, such as cutting speed, feed rate and depth of cut, geometrical and structural properties of tool and sample material properties. In addition to these parameters in the extended cutting speed law, the effect of the Chip Slenderness Ratio (Csr) is taken into consideration as an influential parameter in machining operations, while G is the ratio of the depth of cut to the feed rate. Furthermore, this ratio (Csr) is identified as an equation depending on chip cross-sectional area (A) in extended cutting speed law.

In other words, the greater the Slenderness Ratio (Csr), the greater ratio of cutting depth to the feed rate for the same volumetric chip removal rate. 5 is the most frequently used value in turning operations as a Chip Slenderness Ratio (Csr), in which the selected feed rate is not recommended to be greater than the depth of cut. Therefore, 5 can be taken as a standard value for Csr because of its frequency in practice. Moreover, cutting speed and cutting force consisting of values can be attributed to Csr.

Additionally, the effect of cross-sectional area (A) on the cutting speed is larger than slenderness ratio (Csr). When chip slenderness ratio (Csr) values are between 5:1 and 20:1, there is an increase in cutting speed by 21.5%. The cutting speed diversified about ±9 when the cutting edge angle varied from 0° to 60°. However, the larger cutting edge angle values mostly occur in face milling operations (Kronenberg, 1966).

Chip removal rate is a vital conclusion in machining processes according to machinability criterion. It is derived from multiplying feed rate, depth of cut and cutting speed, which is a vital parameter in manufacturing areas of mass production (Ramana and Kumar, 2018). Higher selected cutting speed, feed rate and depth of cut increase material removal rate linearly. However, the effect of cutting speed is greater than others. An increase in cutting speed at any selected feed rate provides the material removal rate to increase significantly, but it increases slightly with increasing depth of cut, even at higher selected feed rates. Moreover, the most influential parameter on material removal rate is spindle speed, followed by feed rate and depth of cut, respectively (Das et al., 2017), while cutting speed provides higher material removal rate and depth of cut causes tool chatter thus, reducing material removal rate. Furthermore, higher cutting depth creates higher thrust force on cutting tool, pushing the tool radially outwards thereby causing vibrations and leading to decrease in material removal rate. Despite all these, the
Depending on Chip Slenderness Ratio (Csr), the depth of cut is a vital parameter for material removal rate in turning operations (Kumar et al., 2017). However, the effect of cutting speed and depth of cut on feed and radial forces is greater than the feed rate in turning (Ahmed et al., 2015). The residual stress in turning becomes larger with increasing feed rate, but the depth of cut does not have any significant effect (Dahlman et al., 2004).

Chip morphology is influenced by the selected parameters, especially feed rate and depth of cut which cause saw-teeth chip formation as well as, cutting speed and cutting edge radius (Miguelez et al., 2009). Moreover, at higher selected feed rates, removed chip occurs in massive segments. Cutting speed is a vital parameter to cause tool wear and make the machined material harden (Neslusan et al., 2012). Furthermore, greater feed rate and the lower depth of cut provide the lowest in the process temperature. However, higher selected depths of cut cause lower processes temperature. Furthermore, tool temperature decreases at the selected greater feed rate, the lower depth of cut and greater depth of cut, but lower feed rate during the operation (Cui and Guo, 2017). Therefore, with altering the cutting depth, tool life increases, so the consumed energy load of the machine and cutting forces decrease (Sadilek et al., 2016). Additionally, cutting force increases with increasing feed rate, but it decreases with increasing cutting speed (Şeker et al., 2004).

Moreover, selecting cutting depth is a vital phenomenon in turning because it affects the cost of the process (Reddy et al., 1998). Although the depth of cut does not influence tool wear significantly and in continuous turning operations, different depths of cut shorten the tool life (Qin et al., 2014). At lower selected depth of cut values, higher tool wear takes place (Pramanik et al., 2008).

Many different parameters have influence on the residual stress and distortion, making the dimensional precision difficult to control in machining operations. A vital parameter among them is the depth of cut, affecting surface roughness, but there is not a linear proportion between the depth of cut and residual stress (Li et al., 2015). Furthermore, surface roughness is affected by spindle speed and feed rate directly. With increasing feed rate, surface roughness increases linearly, but it decreases with increasing spindle speed (Kumar et al., 2012).

The main purpose of this paper is to investigate the effect of Chip Slenderness Ratio (Csr) on shear, rake, and cutting edge approach angle, by using mathematical and geometrical approach in primary deformation zone.

**GEOMETRY OF ORTHOGONAL CUTTING MECHANISM IN PRIMARY DEFORMATION ZONE**

In this study, orthogonal cutting mechanism is analysed mathematically and geometrically in primary deformation zone. During the modelling process, the effect of Chip Slenderness Ratio (Csr) on shear, rake and cutting edge approach angle was investigated using mathematical and geometrical equation, derived from the orthogonal cutting mechanism according to chip removal rate (Crt). Derived equation was analyzed using Matlab and Minitab software. With the help of these software programs, the relationship between Chip Slenderness Ratio (Csr) – shear, rake and cutting edge approach angles was investigated by gaining graphs.

Material removal rate is a vital conclusion in machining operations, affecting the selected parameters, especially cutting speed, depth of cut and feed rate. Machinability can be evaluated depending on these parameters. In primary deformation zone, when chip is removed from the machined material, the process results are affected by different kinds of parameters according to orthogonal cutting mechanism. One of the most important of these parameters is Chip Slenderness Ratio (Csr) in extended cutting speed law. Therefore, the effect of this parameter (Csr) on shear, rake and cutting edge approach angle was investigated mathematically and geometrically depending on orthogonal cutting mechanism.

**Mechanics of chip formation in turning**

Chip removal rate (Cmr) is obtained by multiplying cutting speed, depth of cut and feed rate. Figure 1 shows that material removal rate can be calculated in the volumetric unit in one cycle of material or in an identified time, such as one second or one minute in turning operations.

Figure 1 shows material removal rate can be calculated geometrically, depending on shear, rake and cutting edge approach angle, associated with depth of cut, cutting speed, namely spindle speed, and feed rate, as in Equation 1 in one cycle of sample material given as:

\[ C_{rm} = a \cdot f \cdot V \]

(1)

Furthermore, cutting speed (V) is calculated depending on spindle speed and diameter of the cylindrical machined sample during one minute time as a period, as seen in Equation 2 and also demonstrated in Figure 1 as (round per minute) rpm:

\[ V_m = \frac{\pi \cdot d \cdot n}{1000} \text{ (m/min)} \]

(2)

Additionally, cutting speed (V) equation can be written depending on the diameter of sample and spindle speed in one second as revolutions in a frequency (Figure 1) and as...
(round per second) rps, can be written as in Equation 3 given as:

\[ V_s = \frac{3.\pi d n_s}{50} \text{ (m/s)} \]  

(3)

Therefore, chip removal rate \( (C_{Rm}) \) is derivable as in Equation 4, depending on cutting speeds in one minute and in one second. This is given as:

\[ C_{Rm} = \frac{\pi d n_s \cdot f \cdot a}{1000} \text{ (mm}^3/\text{min}) \Rightarrow C_{Res} = \frac{3}{50} \cdot \pi d n_s \cdot f \cdot a \text{ (mm}^3/\text{s}) \]  

(4)

**The effect of chip slenderness ratio \( (C_{sr}) \) in primary deformation zone**

Machinability mainly depends on shear and tool cutting edge approach angle, associated with other parameters selected by operators in primary deformation zone. One of most important of these parameters is Chip Slenderness Ratio \( (C_{sr}) \), the ratio of the depth of cut to the feed rate. By means of the orthogonal cutting mechanism, the effect of \( C_{sr} \) on shear, rake and cutting edge approach angles was investigated theoretically using cutting geometry (Figure 2).

The chip removed from the sample \( (t_c) \) can be identified as demonstrated in Equation 5, depending on cutting edge approach angle and feed rate given as:

\[ t_c = f \cdot \cos(\chi) \text{ (mm)} \]  

(5)

Additionally, removed chip \( (t_c) \) is derivable as in Equation 6, depending on uncut chip thickness.

Vertical plane \( (\varepsilon) \), identified with lines in green color is normal to the main plane of the cutting tool. When the removed chip leaves the primary deformation zone, it moves away on the rake face of the tool, which has an inclination, denoting (\( \alpha \)) with the vertical plane \( (\varepsilon) \). The chip is removed from the sample with shear speed \( (V_s) \) and cutting speed \( (V) \); thereafter, it moves away from the zone on the rake face with \( (V_c) \) speed. Additionally, removed chip \( (t_c) \) is derivable as in Equation 6, depending on uncut chip thickness \( (t) \), shear angle \( (\varphi) \), and rake angle \( (\alpha) \) given as:

\[ t_c = \frac{t \cdot \cos(\varphi - \alpha)}{\sin(\varphi)} \text{ (mm)} \]  

(6)

Moreover, as shown in Equation 7, this uncut chip thickness
Chip Slenderness Ratio (Cs) and orthogonal cutting mechanism.

Figure 2: Chip Slenderness Ratio (Cs) and orthogonal cutting mechanism.

The (t) equation can be written down, depending on the shear angle (φ), rake angle (α), and feed rate with the help of orthogonal cutting mechanism as seen in Figure 2 given as:

\[ t = \frac{f \cdot \cos(\chi)}{\sin(\phi)} \cos(\phi - \alpha) \]  

(7)

Chip removal rate is equal to multiplying |AD|, |DC|, and |EA| dimensional sizes according to Figure 2 as demonstrated in Equation 8 given as:

\[ C_{Re} = |AD| \cdot |DC| \cdot |AE| \]  

(8)

|AD|, |DC|, and |EA| magnitudes can be specified depending on uncut chip thickness (t), shear angle (φ), feed rate, cutting edge approach angle (χ), feed rate, the diameter of the machined sample, and spindle speed (\(n_m\) and/or \(n_s\)). In Equations 9, 10, and 11, these dimensional magnitudes are identified:

\[ |AD| = \frac{t \cdot \cos(\chi) \cdot \sin(\phi)}{\cos(\phi - \alpha)} \]  

(9)

\[ |AE| = \frac{\pi \cdot d \cdot n_m}{1000} \Rightarrow |AE| = \frac{3 \cdot \pi \cdot d \cdot n_s}{50} \, (mm) \]  

(10)

\[ |DC| = \frac{a}{\cos(\chi)} \, (mm) \]  

(11)

The volume of removal rate, circumscribed with |ABCDEFGH| points as shown in Figure 2, is calculable by means of the Jacobian method as specified in Equations 12, 13, and 14 given as:

\[ J_{V} = \begin{vmatrix} \frac{d(AD)}{d\phi} & \frac{d(AD)}{dt} & -t \cdot \cos(\phi) & 1 \\ \frac{d(DC)}{d\chi} & \frac{d(DC)}{d\chi} & \sin^2(\phi) & \sin(\phi) \\ \frac{a \cdot \sin(\chi)}{\cos^2(\chi)} & \frac{1}{\cos(\chi)} & & \end{vmatrix} \]  

(12)

\[ C_{Re} = \iiint_{V} J_{V} \, d\phi d\chi dV \]  

(13)
Figure 3: The effect of Chip Slenderness Ratio ($C_{sr}$), a) shear angle ($\phi$), b) cutting edge approach angle ($\chi$), c) rake angle ($\alpha$).

$$C_{sr} = \frac{2}{50} \pi d n_c \left[ \frac{a}{\cos(\chi)} \ln \left( \frac{1}{\sin(\phi) \cos(\phi)} \right) - \frac{t}{\sin(\phi)} \ln \left( \frac{1}{\cos(\chi)} + \tan(\chi) \right) \right]$$

(14)

Chip Slenderness Ratio ($C_{sr}$) is equal to the ratio of the depth of cut to the feed rate as in Equation 15 given as:

$$C_{sr} = \frac{a}{f}$$

(15)

Common solution of Equations 14 and 15 and 16 can be derived:

$$C_{sr} = \frac{2}{50} \pi d n_c f \left[ C_{sr} \frac{a}{\cos(\chi)} \ln \left( \frac{1}{\sin(\phi) \cos(\phi)} \right) - \frac{t}{\sin(\phi)} \ln \left( \frac{1}{\cos(\chi)} + \tan(\chi) \right) \right]$$

(16)

Equalizing Equation 4 to Equation 16, Equation 17 is derivable given as:

$$C_{Rr} = \frac{\cos(\phi - \alpha)}{\sin^2(\phi) \cos(\chi)} \cdot \ln \left( \frac{1}{\cos(\chi)} + \tan(\chi) \right)$$

(17)

RESULTS AND DISCUSSION

The relationship between $C_{sr}$ and $\phi$, $\chi$, $\alpha$ angles

Chip Slenderness Ratio ($C_{sr}$) is computable depending on shear angle ($\phi$), cutting edge approach angle ($\chi$), and rake angle ($\alpha$) using Equation 17. Figure 3a, b and c shows the
relationship between Chip Slenderness Ratio ($C_{sr}$) and shear ($\phi$), cutting edge approach ($\chi$) and rake ($\alpha$) angles.

Figure 3a, b and c shows that according to Equation 17 using Matlab software, the graphs of $C_{sr}$ and $\phi$, $\chi$, $\alpha$ were achieved. For the algorithm, applying Matlab software, shear angle ($\phi$), cutting edge approach angle ($\chi$), and rake angle ($\alpha$) were selected between 0° to 45°, 0° to 45° and 0° to 20°, respectively. Equalizing of $C_{sr}$ to 5 is standard in practice machining operations, especially in turning (Kronenberg, 1966). Figure 3a shows the graphs of the relationship between shear angle ($\phi$) and $C_{sr}$. According to this graph, shear angle ($\phi$) is specified in radian unit. If we multiply values on the shear angle ($\phi$) axis with 180/π, the angle values are obtainable in degree unit.

According to graph in Figure 3a, there is an inversely proportional relationship between the shear angle ($\phi$) and Chip Slenderness Ratio ($C_{sr}$). According to the graph, the optimum Chip Slenderness Ratio ($C_{sr}$=5) shear angle is equal to about 15°. When the shear angle ($\phi$) decreases $C_{sr}$ value increases linearly. When shear angle values are between 15° and 45°, ($15^\circ \leq \phi \leq 45^\circ$), Chip Slenderness Ratio ($C_{sr}$) is smaller than 5 ($C_{sr}$≤5). With changing shear angle values between 0° to 45°, associated with the rake ($\alpha$) and cutting edge approach ($\chi$) angles, $C_{sr}$ takes values smaller than 40 and bigger than 0, (0≤$C_{sr}$≤40). Although there is an inversely proportional relationship between $C_{sr}$ and $\phi$; there is also a directly proportional relationship between cutting edge angle ($\chi$) and Chip Slenderness Ratio ($C_{sr}$).

However, the inclination of the curve of this relationship changes associated with the shear angle ($\phi$) and rake angle ($\alpha$) (Figure 3b). For the standard $C_{sr}$ values smaller than 5, cutting edge approach angle ($\chi$) takes values approximately bigger than 6°. At smaller selected cutting edge approach angle ($\chi$), $C_{sr}$ equal values are bigger than 5 (Figure 3b).

Contrary to shear ($\phi$) and cutting edge approach ($\chi$) angles, rake angle ($\alpha$) has an inconsiderable impact on Chip Slenderness ratio ($C_{sr}$). Figure 3c shows that with changing rake angle ($\alpha$) between 0° to 20°, $C_{sr}$ changes about 1 unit. This result shows that rake angle ($\alpha$) does not have insignificant effect on $C_{sr}$ in primary deformation zone. Because rake angle ($\alpha$) affects the moving of removed chip on the rake face of the cutting tool, it does not have any significant effect in primary deformation zone.

**The relationship between $C_{sr}$ and $\phi$, $\chi$, $\alpha$ angles**

Furthermore, the relationship between Chip Slenderness Ratio ($C_{sr}$) and shear ($\phi$), cutting edge approach ($\chi$), and rake ($\alpha$) angles were investigated by means of Minitab 17 software program. This relationship is shown in Figure 4. This relationship shows an alteration compatible with curves, achieved by using the Matlab software. In Figure 4, the relationship between $C_{sr}$ and $\phi$ is demonstrated with
three different kinds of curves and also the relationship of $\alpha$ with two kinds of curves, while $\varphi$ is shown with only one curve. As interpreted under 3.1 title, with increasing shear angle ($\varphi$), Chip Slenderness Ratio ($C_{sr}$) decreases, while it increases with increasing cutting tool approach angle ($\chi$), but there is no insignificant rake angle ($\alpha$) on Chip Slenderness Ratio ($C_{sr}$).

Conclusions

Although Chip Slenderness Ratio ($C_{sr}$) has a vital influence on machining operations in extended cutting speed, it has been scarcely studied by researchers in literature. It has a vital impact on the shear angle ($\varphi$) and cutting edge approach angle ($\chi$) in primary deformation zone, though it has an insignificant effect on rake angle ($\alpha$).

There is an inversely proportional relationship between the shear angle ($\varphi$) and Chip Slenderness Ratio ($C_{sr}$), while there is a direct proportion relationship with cutting edge approach angle ($\chi$).

However, rake angle ($\alpha$) has an insignificant impact on Chip Slenderness Ratio ($\chi$) because rake angle ($\alpha$) affects only the moving of removed chip on the rake face of the cutting tool in secondary deformation zone.

The optimum shear angle ($\varphi$) is about 15° according to the optimum Chip Slenderness Ratio ($C_{sr}$), which is equal to 5. Moreover, increases in shear angle ($\varphi$) cause decreasing $C_{sr}$ smaller than 5, and invertibility is also valid. However, with increasing cutting edge approach angle, $C_{sr}$ increases linearly, but the inclination of the curve decreases with decreasing $\chi$, associated with other selected parameters.

REFERENCES


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