Survival analysis applied and studied by multidimensional time model for probability cumulative function along with different methods for approximation of survival function of random trees and forests including maximum entropy and saddle point approximations

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ABSTRACT

The new method is based on changes of cumulative distribution function in relation to time change in sampling patterns. Multidimensional time model for probability cumulative function can be reduced to finite-dimensional time model, which can be characterized by Boolean algebra for operations over events and their probabilities and index set for reduction of infinite dimensional time model to finite number of dimensions of time model considering also the fractal-dimensional time arising from alike super symmetrical properties of probability. It is based on the properties of composition of Brownian motion processes applied through application of Boolean prime ideal theorem and Stone duality. This model can be applied to Survival analysis that is, by itself a modeling of time to event data. Survival analysis that does not use ordinary regression analysis is a good place for demonstration of James-Stein estimators for multidimensional exponential family, types of multivariate Gamma Distribution with different methods for approximation of Survival Function of Random Trees and Forests including Maximum Entropy and Saddle Point Approximations.

Keywords: Multidimensional time model, time to event, periodicity.

INTRODUCTION

John Graunt developed the first “life table” that are used by actuaries some 110 years before Lambert’s approximation given as: \((1 - x/96)^2 \cdot 0.6176(\exp(-x/31.682) - \exp(-x/2.43114))\). The basic life-table methodology in modern terminology amounts to the estimation of a survival function from life times with delayed entry, or left truncation and right censoring. This was known before 300 years, and explicit parametric models since the linear approximation of de Moivre some 50 years before Lambert’s work.

Survival analysis is generally defined as a set of methods for analyzing data where the outcome variable is the time until the occurrence of onetime event of interest. The event can terminate the process, or repeat it. The time until event is called survival time is subject to time-scale (days, weeks and years, etc). The dependent variable in survival analysis is composed of two parts: one is the time to event and the other is the event status, which records if the event of interest occurred or not.

Survival function is defined as:

\[ S(t) = \Pr(T > t) \]  \hspace{1cm} (1)

With:

\[ F(t) = \Pr(T \leq t) = 1 - S(t) \]  \hspace{1cm} (2)

as the lifetime distribution function.

If \( F \) is differentiable pdf \( f(t) = F'(t) \) is sometimes called
the event density:

$$s(t) = S'(t) = -f(t) \quad (3)$$

In other fields, such as statistical physics, the survival event density function is known as the first passage time density.

In contrast to simple regression models, survival methods can use uncensored and censored data. If there is no censoring, standard regression procedures usually are used. However, these may be inadequate even in fitting Exponential, Weibill and extreme value distributional models because:

1) Survival time should be positive and has a skewed distribution;  
2) Focus in many problems should be made more on the probability of surviving past a certain point in time than on the expected survival time and the former can follow a different model and require different fitting;  
3) The hazard function in survival analysis gives more insight into the failure mechanism that can be applied in its turn to linear regression.

Hazard rate function at time x is defined as:

$$z(x) = \frac{f(x)}{1-F(x)} \quad (4)$$

**Types of right-censoring**

The different types of right-censoring are:

1) Fixed type I censoring which occurs when a study is designed to end after C years of follow-up. In this case, everyone who does not have an event observed during the course of the study is censored at C years.  
2) Random type I censoring, where the study is designed to end after C years, but censored subjects do not all have the same censoring time. This is the main type of right-censoring of interest.  
3) Type II censoring, where a study ends when there is a prespecified number of events.

We can estimate the survival distribution by making parametric assumptions:

- Exponential;  
- Weibull;  
- Gamma;  
- log-normal.

There are several other statistical phenomena that can be mentioned:

**Theorem 1**: Let a, b > 0 be given constants and let U, V be iid uniform (0, 1] random variables (Berman, 1971). Then, conditioned on U^a + V^b ≤ 1, the random variable U^a is beta (a, b +1) distributed.

**Theorem 2**: Let a, b > 0 be given constants and let U, V be iid uniform (0, 1] random variables (Johnk, 1964). Then, conditioned on U^a + V^b ≤ 1, the random variable [U^a/(U^a + V^b)] is beta (a, b) distributed.

**Periodicity of time series**

The periodicity, that is, the periodic fluctuations of time series can be tested with the serial correlation coefficient, but only after proving that there is no definite time trend. The serial correlation coefficient (rs) is defined as:

$$r_s = \frac{\Sigma'(x_{i+1})/\Sigma'x^2}{\Sigma x^2} \quad (5)$$

For observations at time i and i + 1, if rs is significant and a time trend is absent, then one can conclude that there must be a periodicity.

Censoring is a form of missing data problem which is common in survival analysis. Clinical trials that are designed with a minimum follow-up time lack reliability because of patients with very short follow-up time. Other types of censoring in quality-adjusted lifetime and different medical costs contribute to different statistical phenomena.

For cumulative hazard function:

$$W(t) = \int_0^T z(u)du \quad (6)$$

Where S(t) = e^{-W(t)}.

If U denote the censoring time, then (T, U) are referred to as latent data and (X, Y) are observed data with X=min(T, U) and Y=1(T≤U). It is already more than 40 years from the works of Tsiatis and Peterson, who succeeded to show that observation of (X, Y) does not allow to identify or consistently estimate z(t) or S(t).

An alternative to building a single survival tree is to build many survival trees where each tree is constructed using a sample of the data and average trees to predict survival; the random forest survival model gives more accurate predictions of survival than the Cox PH model. The Cox PH regression model is a linear model. It is similar to linear and logistic regressions. Specifically, these methods assume that a single line, curve, plane, or surface is sufficient to separate groups or to estimate a quantitative response (survival time).

All the aforementioned discussion reminds of using trees...
or forests with possibly mixtures of exponential and Gamma functions with possibly Bayesian priors.

**THE EXPONENTIAL AND GAMMA DISTRIBUTIONS**

The exponential distribution occurs naturally when describing the lengths of the inter-arrival times in a homogeneous Poisson process, which is, following W. Feller v.2 p.9:

\[ f_i(t) = e^{-\lambda t} \frac{\lambda^t}{t!} \quad (7) \]

This was first discovered by John Michell some 70 years before Poisson with consideration of distribution for the random scattering of points in the region without introduction of Poisson distribution. Poisson process with \( \lambda = a \) would become Poisson distribution and it was previously obtained by Abraham de Moivre as a limiting case of binomial with \( \lambda = np \).

It looks like that the most reasonable is to use some mixture of Gamma distribution functions for some small time analysis that we can suppose to correctly fit the model at least piecewise and possibly then extend it in some way for a long range of time. That does not imply to that end and there would be exact fitting of the model, however, we would be far away from simulating the fit, as in regression models and much more close to modeling the process. However, the resembling concept may be learned from Time-dependent Effects, where the piece-wise exponential scheme contributes to the introduction of non-proportional hazards or time-varying effects with the effects varying only at interval boundaries.

Let \( T_1, T_2, \ldots, T_i \) be iid rv \( E(\lambda), \lambda > 0 \), where \( E(\lambda) \) is for exponential distribution function then:

\[ \sum_{i=1}^{n} T_i \sim \Gamma(n, \lambda) \quad (8) \]

Let \( T_1, T_2, \ldots, T_i \) be iid rv \( \Gamma(a_i, \lambda), a_i, \lambda > 0 \), where \( \Gamma(a_i, \lambda) \) is for gamma distribution function, then:

\[ \sum_{i=1}^{n} T_i \sim \Gamma\left(\sum_{i=1}^{n} a_i, \lambda\right) = \frac{\lambda^{\sum_{i=1}^{n} a_i} t^{\sum_{i=1}^{n} a_i - 1} e^{-\lambda t}}{\Gamma\left(\sum_{i=1}^{n} a_i\right)} \quad \text{if}(t) \text{ for } t > 0, \]

where \( \Gamma(x) = \int_{0}^{\infty} t^{x-1} e^{-t} dt \) and \( \sum_{i=1}^{n} a_i = m \) \( (9) \)

Let \( T_1, T_2, \ldots, T_i \) be iid gamma rv with \( T_i \) having parameters \( a_i \) and \( \lambda_i \) for \( S=\sum_{i=1}^{n} T_i \) can be used for convolution Theorem (Feller):

\[ S \sim f(t; m_1, \lambda_1) \ast f(t; m_2, \lambda_2) \ast \ldots \ast f(t; m_n, \lambda_n) \quad (10) \]

Where \( \ast \) denotes convolution.

\[ G(t) \ast H(t) = \int_{0}^{t} G(t - s)H(s)ds \quad (11) \]

The variety of different methods and types of bivariate exponential and Gamma distribution allow flexible approach for the evaluation of data. There are at least 20 different procedures only for Poisson distribution for finding confidence intervals.

**Variety of methods of linear combinations and ratios of random variables**

Five methods of the theory and applications of linear combinations and ratios of random variables are highlighted as:

1) Ratios of normal random variables appear as sampling distributions in single equation models and simultaneous equations models, as posterior distributions for parameters of regression models;

2) Weighted sums of uniform random variables;

3) Ratio of linear combinations of chi-squared random variables multivariate linear functional relationship model Sums of independent gamma random variables;

4) Linear combinations of inverted gamma random variables for the Behrens-Fisher problem and variance components in balanced mixed linear models Beta distributions and their linear combinations;

5) Linear combinations of the form \( T = a_1 t_1 + a_2 t_2 \) where \( t_1 \) denotes the Student \( t \) random variable based on \( f \) degrees of freedom and weighted sums of the Poisson parameters.

**Five types of the bivariate exponential distribution**

The Five types of the bivariate exponential distribution include:

1) \( \int_{x}^{\infty} \left( \int_{y}^{\infty} f(x, y)dy \right) dx = F(x; y) = 1 - \exp(-a_1 x - a_2 y - a_3 \max(x, y)), x, y \geq 0 \) \( (12) \)

for Marshall and Olkin model.

2) \( f(x,y)=1-\exp(-x) - \exp(-y) + \exp(-x-y-axy); \quad x,y \geq 0, 0 \leq a \leq 1 \) \( (13) \)

for Gumbel model.

3) \( X = U_1^2 + U_2^2, Y = U_3^2 + U_4^2 \) with \( (U_1, U_2) \) independent from \( (U_3, U_4) \), but have the same joint normal with zero means, variances \( \frac{1}{2} \), and corr \( r(0 \leq r \leq 1) \) for Moran model.

4) For Freund model components 1 and 2 are dependent in
that a failure of either component changes the parameter of the life distribution of the other component.

5) Block and Basu considered a bivariate distribution whose marginals are mixtures of exponentials and having an absolutely continuous joint distribution.

Five types of bivariate gamma distributions

In univariate case, the gamma distributions are generalization of Erlang distribution, which is the sum of i.i.d and exponentially distributed random variables:

- McKay's bivariate gamma distribution given by the joint pdf:
  \[ f(x, y) = \frac{(a + q)}{\Gamma(p) \Gamma(q)} x^{q-1}y^{p-1} \exp(-ay) \]  
  for \( y > x > 0, \ a > 0, \ p > 0 \text{ and } q > 0. \)

- Cherian's bivariate gamma distribution given by the joint pdf:
  \[ f(x, y) = \frac{(\exp(-x-y))}{\Gamma(p) \Gamma(q)} \Gamma(p+q)(x+y)^{p+q-1} \exp(-xy) \]  
  for \( x > 0, y > 0, \ a > 0, \ p > 0 \text{ and } q > 0. \)

- Kibble's bivariate gamma distribution given by the joint pdf:
  \[ f(x, y) = \frac{(xy)^{1/2}}{\Gamma(1-p) \Gamma(1-q)} \exp(-\frac{x+y}{y}) \exp(-\frac{1-ho}{1-ho}) \exp(-\frac{x+y}{y}) \]  
  where \( l(i) \) denoted modified Bessel function of the first kind of order \( i. \)

- The Beta Stacy distribution is given by the joint pdf:
  \[ f(x, y) = \frac{c}{\mathcal{B}(a,b) \mathcal{B}(p,q)} x^{y-1} \exp(-\{y/a\}^p) \]  
  where \( \mathcal{B}(\cdot) \) is Beta distribution function for \( y > x > 0, \ a > 0, \ b > 0, c > 0, \ p > 0 \text{ and } q > 0. \)

- Becker and Roux's bivariate gamma distribution given by:
  \[ f(x, y) = \frac{\beta' \alpha^a}{\Gamma(\alpha) \Gamma(\beta)} \exp(\beta' (y - x) + \beta \chi) x^{\alpha-1} \exp(\beta' y - (\alpha + \beta' \chi) \chi), \text{if } y > x > 0, \]
  or
  \[ \frac{\alpha' \beta^b}{\Gamma(\alpha) \Gamma(\beta)} (\alpha' (x - y) + \alpha y)^{y-1} \exp(-\alpha' x - (\alpha + \beta - \alpha') \chi), \text{if } y > x > 0, \]
  for \( x > 0, y > 0, \ a > 0, \ b > 0, \ a' > 0, \ b' > 0 \text{ and } \beta' > 0. \)

DIFFERENT METHODS OF APPROXIMATING PROBABILITY DISTRIBUTIONS

Method of maximum entropy

According to the Principle of Maximum Entropy the best approximating model probability distribution with exactly known prior data is with the largest entropy.

Entropy of a discrete rv \( X \) is defined as:

\[ H(X) = E(-\ln P(X)) = E(I(X)) = \sum_{i=1}^{n} P(x_i) I(x_i) = - \sum_{i=1}^{n} P(x_i) \ln (P(x_i)) \]  

(19)

Where \( I(X) \) represents information.

Conditional Entropy of a rv \( X \) conditional on \( Y \) is:

\[ H(X/Y) = \sum_{i,j} P(x_i,y_j) \ln \frac{P(x_i)}{P(x_i,y_j)} \]  

(20)

For discrete rv, \( X \) with values in \( \{x_1, x_2, ..., x_n\} \) the information \( I \) has the form of \( m \) constraints on the expectations of the functions \( f_k \). The probability distribution subject to constrain is:

\[ P(x_i/I) = \frac{\sum_{i=1}^{n} P(x_i)}{Z(\lambda_1, ..., \lambda_m)} \]  

(21)

Where the normalization constant:

\[ Z = \sum_{i=1}^{n} \exp(\lambda_1 \mathcal{F}_1(x_1) + ... \mathcal{F}_m(x_m)) \]  

(22)

\[ F_k = \frac{\partial}{\partial \lambda_k} \ln Z(\lambda_1, ..., \lambda_m) \]  

(23)

Where there are Lagrange multipliers and the system of equations can be solved with numerical methods.

Conditional or Bayesian model would become:
\[ p(x_i | Y) = \frac{\exp \left( \sum_k \lambda_k f_k(x,y) + \cdots + \lambda_m f_m(x,y) \right)}{\sum_{x \in X} \exp \left( \sum_k \lambda_k f_k(x,y) + \cdots + \lambda_m f_m(x,y) \right)} \quad (24) \]

**Method of saddle point approximations**

For cumulant generating function \( K(t) = \log(M(t)) \), where \( M(t) \) is the moment generating function, then the saddle point approximation to the PDF and CDF of a distribution is given by:

\[
\hat{f}(x) = \frac{1}{\sqrt{2\pi K'(\hat{\theta})}} \exp \left( K(\hat{\theta}) - \hat{\theta} x \right) \quad (25)
\]

\[
\hat{F}(x) = \Phi(\hat{\omega}) + \phi(\hat{\omega}) \left( \frac{1}{\hat{\omega}} - \frac{1}{\hat{\theta}} \right) \quad \text{for} \ x \neq E(x) \quad (26)
\]

\[
\frac{1}{2} + \frac{K''(0)}{6\sqrt{2\pi K(0)^{3/2}}} \quad \text{for} \ x = E(x) \quad (27)
\]

Where \( \hat{\theta} \) is a solution to:

\[
K'(\hat{\theta}) = x, \quad \hat{\omega} = \text{sgn} \hat{\theta} \sqrt{2(\hat{\theta} x - K'(\hat{\theta}))} \quad (28)
\]

And \( \hat{\omega} = \sqrt{K''(\hat{\theta})} \quad (29) \]

This can be applied to hazard function that can be approximated by:

\[
\hat{\Lambda}(x) = \frac{\hat{f}(x)}{1 - \hat{F}(x)} \quad (30)
\]

**MULTIDIMENSIONAL TIME MODEL FOR PROBABILITY CUMULATIVE FUNCTION**

However, it also makes reasonable application of multidimensional cumulative distribution function that can be reduced to finite-dimensional time model, which can be characterized by Boolean algebra for operations over events and their probabilities and index set for reduction of infinite dimensional time model to finite number of dimensions of time model considering the fractal-dimensional time arising from alike supersymmetrical properties of probability. The new method is based on properties of Brownian motion and philosophically intended to reach high level of precision.

Multidimensional cumulative distribution function can be reduced to finite-dimensional time model, which can be characterized by Boolean algebra for operations over events and their probabilities and index set for reduction of infinite dimensional time model to finite number of dimensions of time model considering the fractal-dimensional time arising from alike supersymmetrical properties of probability. The new method is based on properties of Brownian motion. This can lead to various applications for parameter evaluation such as the extension of the original one-dimensional Kramers model for application of Brownian motion to chemical reactions to multidimensional consideration.

**First approach to multidimensional time model for probability cumulative function through Kramers turn-over problem in the theory of velocity of chemical reactions**

Consider first the mathematical structure of the models of Boltzmann type kinetic equations for reacting gas mixtures for particles undergoing inelastic interactions with reactions of bimolecular and dissociation-recombination type is very complicated, due to the collisional operators that usually in the full Boltzmann equations are expressed by 5-fold integrals. Consequently, direct numerical applications of these models present several computational difficulties. The search for the simpler solution had its long way till the introduction of the equation for the Brownian motion by Albert Einstein.

However, using the theory of Brownian motion for the velocity (rate) of chemical reactions Bohr, Kramers and Slater used only one-dimensional (1D) model for the Kramers turn-over problem, that is, obtaining a uniform expression for the rate of escape of a particle over a barrier for any value of the external friction until it was corrected by Grote-Hynes theory 40 years later, with new improvements following after 6 years by Mel’nikov and Meshkov (MM). There are certainly other theories followed, all of which distinguish 1D approach from 2D, 3D and multiD approaches.

It is important and very interesting to consider such point that Kramers in his original work had as possibility that multidimensional pattern could be related to time dimensions, as he based his introduction theory of Brownian motion on the Einstein’s pattern he considered a range of time intervals \( \tau \). His discussion of the possibility of a term proportional to \( B_\tau \) in the expression for moments of Brownian motion \( B_\tau(n>1) \) is related to the fact that the values, which \( X \) takes at moments \( t_0, t_2, \ldots, t_n \), which lie sufficiently close together are no longer independent. Moments of Brownian motion \( B_\tau(n>1) \) in fact are represented by a volume integral given as:
\[ \int \cdots \int X(t_1)X(t_2) \cdots X(t_n) \, dt_1 dt_2 \cdots dt_n \]

over an n-dimensional cube; the contribution to this integral due to a narrow cylinder extending along the diagonal \( t_1 = t_2 = \ldots = t_n \) may give a term proportional to \( T \).

**Second approach to multidimensional time model through cumulant functions and time series analysis**

To strengthen these notions consider cumulants properties for time series analysis that provide measure of Gaussianity. If r.v. \( X \) is normal, then, \( \text{cum}_k(X) = 0 \) for \( k \geq 2 \), where \( \text{cum}_k \) denotes the joint cumulants of \( X \) with itself \( k \) times.

For simplicity, consider seq of iid \( X_i \) with all moments and \( \text{E}(X_i) = 0 \) and \( \text{var}(X_i) = 1 \), then, for \( S_n = \sum X_i / \sqrt{n} \), cumulant

\[
\text{cum}_k(S_n) = n \text{cum}_k(X) / n^{k/2} \text{ that tends to 0 for } k \geq 2, \text{ as } n \text{ tends to infinity}, \text{ so } S_n \text{ has a limiting normal distribution. For time series analysis the moment function: } \text{E}(X(t+u_1) \cdots X(t+u_k)) \text{ would not depend on } t, \text{ and on the short time interval centered at point of time } t \text{ can be approximated by normal distribution.}
\]

**Third approach through associated random variables**

Additional to the Brownian motion considerations in the theory of chemical reactions and time series analysis for cumulant functions, the same results can be obtained from the consideration of associated random variables.

**Definition b891**: For \( n \geq 1 \), the set of \( X_i \) is said to be associated, if for all given real-valued functions \( g_i \) that are increasing in each component when the other components are held fixed. The inequality

\[
\text{E} \left( \prod_i g_i(X) \right) \geq \prod_i \text{E}(g_i(X)) \text{ holds, or equivalently, Corr} (g_i(X), g_b(X)) \geq 0.
\]

**Theorem 3**: (a) A set consisting of a single random variable is a set of associated random variables; (b) Independent random variables are associated random variables; (c) A subset of a set of associated random variables forms a set of associated random variables and (d) Increasing functions of associated random variables are associated random variables.

**Proposition 1**: Therefore, the process \( X(t) \) with aforementioned properties can be represented by composition of Brownian motion processes in finite-dimensional time model.

Next, consider Stone representation of Boolean algebra, which is represented by an algebra with known axioms for Boolean algebra and can be characterized by quadruplets \( B = \langle X, 0, *, \rightarrow, \rangle \), where 0 is an element from a set X, and * is a binary operation and \( \rightarrow \) is an unary operation, which would be a Boolean algebra with 1 as a unit on the operations \( \Lambda, V \) and \( \rightarrow \). Besides that it has four unary operations, two of which are constant operations, another is the identity and negation and besides the number of \( n \)-ary operations, the number of the dimensions that infinite-dimensional model can be reduced to through application of Boolean prime ideal theorem and Stone duality can be indexed by an index set.

**Proposition 2**: Multidimensional time model for probability cumulative function can be reduced to finite-dimensional time model, which can be characterized by Boolean algebra for operations over events and their probabilities and index set for reduction of infinite-dimensional time model to finite number of dimensions of time model considering the fractal-dimensional time arising from alike supersymmetrical properties of probability.

**EXAMPLES OF SCIENTIFIC APPLICATION**

**Multiple testing dependence**

**Definition 2**: For the model in matrix form: \( Y = \text{CS}(X) + E \) the problem of testing \( m \) hypotheses of the form: \( H_0: c_i \in \Omega_0 \text{ vs. } H_1: c_i \in \Omega_1 \) has typically been defined in terms of \( P \) values or test statistics resulting from multiple tests.

**Definition 3**: Population-level multiple testing dependence exists if \( \text{P}(y_1, y_2, y_n | X) \neq \text{P}(y_1 | X) \times \text{P}(y_2 | X) \times \cdots \times \text{P}(y_n | X) \). Correspondingly, estimation-level multiple testing dependence exists if \( \text{P}(y_1, y_2, y_n | \hat{C}, S(X)) \neq \text{P}(y_1 | \hat{C}, S(X)) \times \text{P}(y_2 | \hat{C}, S(X)) \times \cdots \times \text{P}(y_n | \hat{C}, S(X)) \), where \( \hat{C} \) denotes the estimator of \( C \). The dependence is equivalent to dependence of the rows of \( E \), or the rows of residual matrix.

**Proposition 3**: Suppose that for each \( e_i \) in the aforementioned model, there is no Borel measurable function, \( g \) such that \( e_i = e_i | g(e_1, \ldots, e_{i-1}, e_{i+1}, \ldots, e_n) \) almost surely. Then, there exist matrices: \( D_{n \times n} G_{n \times k} (r \leq K) \), and \( U_{n \times k} \) such that \( Y = \text{CS} + DG + U \) where the rows of \( U \) are jointly independent random vectors so that \( \text{P}(u_1, u_2, \ldots, u_n) = \text{P}(u_1) \times \text{P}(u_2) \times \cdots \times \text{P}(u_n) \), and for all \( i = 1, 2, \ldots, N \), \( u_i \neq 0 \) and \( u_i = h_i(e) \) for a non-random Borel measurable function \( h \).

**Corollary 1**: Under the assumptions of this Proposition 3, all population level multiple testing dependence is removed when conditioning on both \( X \) and a dependence
kernel \( G \). Similar result holds for estimation-level multiple testing dependence when conditioning on \( X \) and a dependence kernel \( G \) and estimators \( \hat{C} \) of \( C \) and \( \hat{D} \) of \( D \).

REFERENCES

De Moivre A (1725). Annuities upon Lives: or, The Valuation of Annuities upon any Number of Lives; as also, of Reversions. To which is added, An Appendix concerning the Expectations of Life, and Probabilities of Survivorship. Fayram, Motte and Pearson, London.


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