Modelling of air boundary layer around the grinding wheel

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ABSTRACT

When a grinding wheel rotates in the static air, a boundary layer of air is formed around it which obstructs the entry of coolant into the grinding zone. The modelling of this air boundary layer is a complex process. In most of the cases, the ideal conditions are considered for modelling it. In this paper, the real field conditions such as spouting and the surface roughness effect of a grinding wheel on the air flow was also considered along with the other prevailing conditions for modelling the pressure field. Therefore, it can be utilized to predict the air pressure around a porous and non-porous wheel and this prediction can help in making the strategy of impinging the cutting fluid jet into the grinding zone. The models are developed initially from the Navier-Stokes (N-S) equation. Further, a modified model is proposed by adding a correction factor to compensate the additional effects which contribute to the formation of air boundary layer. The Flower Pollination Algorithm (FPA) was employed to optimize the parameters of the correction factor. The models generated were found consistent with the experimental results.

Keywords: Modelling, air boundary layer, grinding wheel, solid wheel, spouting, compensating air, Navier-Stokes, correction factor, FPA.

INTRODUCTION

Grinding is a finishing or semi-finishing machining process associated with a large amount of heat generation compared to other machining processes. The controlling of the heat production and reduction of the thermal damage is a challenge in this machining process (Malkin and Guo, 2007).

Conventionally, grinding fluid is applied to minimize the effect of the high temperature (Sales et al., 2001), but when a wheel rotates, a boundary layer of air is formed around the wheel due to viscous effect which obstructs the flow of coolant jet to reach deep into the grinding zone and thus, hampers the process performance. Due to this, the thermal damage, inaccuracies in dimension, residual stresses, micro-cracks etc are observed in the component at the final stage of production (Majumdar et al., 2016, 2015; Mandal et al., 2010; Zhang et al., 2016).

The presence of this air layer has a significant effect on the product quality and productivity. Therefore, the study of this air layer formation is required so that appropriate strategy can be made to remove its ill effects.

Gviniashvili et al. (2005) investigated that the flow rate of the grinding fluid through the contact zone depends upon four factors, such as the pressure of the fluid in the grinding zone, the flow rate of the coolant, wheel velocity and fluid density. It is found that the theoretical useful flow rate of the fluid increases when the airflow around the impervious wheel is restricted. Ebrell et al. (2000) observed that the momentum of the reverse air flow of the air boundary layer at the grinding zone does not allow the grinding fluid to pass beneath the grinding wheel. For overcoming the effect of boundary air, they employed the fluid delivery nozzle in three different orientations and angular positions. The horizontal position of the nozzle is found to be more beneficial due to less consumption of grinding power than the tangential and angular alignments.

The aerodynamic baffle was used to inhibit the air flow in order to enhance the grinding performance. A 74.5%
Application of scraper board and pneumatic barrier in recent past was also used to remove the consequence of air barrier formation, but the formation of air layer again crops up within a short distance after the scraper board or pneumatic barrier (Majumdar et al., 2017; Mandal et al., 2015).

Many researchers have understood that velocity of the cooling jet is required to increase up to the level of the peripheral velocity of the wheel to combat the pressure of the rotating air (Gviniashvili et al., 2004), but this remedy increases the production cost. Therefore, in order to design the proper remedial mechanism to abolish the effect of this air barrier, a thorough knowledge about its formation is required.

In theoretical studies of the air boundary based on Navier-Stokes (N-S) equation, consideration of all the real-field effects becomes difficult. Hence, the model does not match well with the experimental results. Li et al. (2013) developed a theoretical model of air flow from N-S equation considering the centrifugal effect along with the shear stress term which can predict only the pressure difference between inside and outside of the air bond formed around the wheel.

Further, the model developed was not validated with the experimental data, but to the knowledge of the author, no model has been found established from the N-S equation considering the wheel surface roughness and the spouting effect to predict the air pressure more accurately at every point within the flow region in the radial direction of the wheel. Therefore, in this paper, a model was proposed where the surface roughness of the wheel and the spouting effect of the air were considered with the model developed from the momentum equation that describes the flow field. Vita et al. (2012) assumed a constant pressure across the air boundary in order to find the instantaneous thickness of the air film.

The present model is able to predict the change in pressure of air at all the points at radial direction of the wheel within the flow field. Also, application of FPA in deriving the model of air flow around a grinding wheel is also novel in its kind.

BOUNDARY LAYER FORMATION ANALYSIS

When a wheel rotates in still air due to friction between the wheel surface, air and the viscous effect, this air is carried along. This promotes the formation of the boundary layer around a rotating wheel or disc (Figure 1). The air layer which is nearer to the solid surface may experience the same velocity of the wheel due to no-slip condition. Due to the presence of this layer, it becomes difficult to feed the process fluid into the grinding zone. Different models presented are attempts to reveal some insights about it.

CFD MODELLING OF AIR FLOW FIELD

A CFD analysis is carried out to examine the formation of air layer around a grinding wheel. The CFD analysis by COMSOL 5.0 Multiphysics software shows such development of boundary layer when a wheel is rotated at 2880 rpm, the density and dynamic viscosity of air is taken as 1.205 kg/m³ and 12.53 × 10⁻⁶ Pa·s respectively (Figure 2). The radius of the wheel and control volume is taken as 0.1 m and 1 m³ respectively. The surface roughness and porosity of the wheel are neglected in this study. It is
observed from the model that a layer of air is formed around the wheel and the thickness of the boundary layer is less than the radius of the wheel. The velocity of the boundary air layer is found to decrease gradually towards radially outward direction.

**EXPERIMENTAL STUDY**

From CFD analysis (Figure 2), the existence of air flow field around a grinding wheel is observed. Correspondingly, the velocity and pressure are found to decrease at radial outward direction. Hence, experimental soundings are required to investigate the characteristics of this flow domain. Experiments are conducted on an alumina grinding wheel of the specification: AA46/54K5V8 and size: \( \Phi 200 \times 20 \times \Phi 31.75 \) (mm) fitted in a horizontal surface grinding machine at 2880 rpm. A cast iron wheel of the same dimension of the grinding wheel is also employed for observing the air flow field around it. A Prandtl type Pitot tube is chosen with coefficient 0.983 for measuring the air pressure by the deflection of the water column in a U-tube manometer. The pitot tube of diameter 6 mm is initially placed at 0-0 position and the axis placed 3.5 mm away from the grinding wheel surface (Figure 3). Thereafter, it is placed at 2 mm succession on either side of...
the 0-0 position along the thickness of the wheel. And from each of these locations, the Pitot is then moved in negative y-direction or in the radial outward direction gradually. The readings of the air pressure are noted at 2 mm succession in radial outward direction till it becomes zero.

Modelling of air flow field from Navier-Stokes equation

The surface forces play an important role in the rotation of the air boundary around the grinding wheel. Surface forces (pressure, friction) including both normal and tangential forces are produced when a solid surface meet other particles (Fox et al., 2004). It acts across a surface of a material body. When a grinding wheel rotates, air particles meet it. Hence, the surface forces are generated. The air is carried along the wheel by the surface forces. Again, the centrifugal effect on air causes the fluid particles to leave the wheel in the radial outward direction. Hence, both the surface and centrifugal forces are considered in the calculation of air pressure around the wheel (Figure 4). The effect of gravitational and body forces are neglected in this study.

For deriving the differential equation of air pressure around the grinding wheel from the Navier-Stokes equation a differential control volume is considered in the Cartesian co-ordinate. It is assumed that the flow is incompressible and the change in pressure in z-direction is constant. Figure 5 shows the forces acting on a differential control volume in the flow domain. The centrifugal force (dFcen) on the fluid acts in the radial direction of the wheel and hence, it is considered along with the surface force (dFs) in y-direction. In tangential direction, only surface force is considered. Hence, the forces acting on the control volume per unit mass in x and y-direction is given by:

\[
\begin{align*}
\text{d}F_x &= \text{d}F_s_x \\
\text{d}F_y &= \text{d}F_s_y + \text{d}Fcen
\end{align*}
\]

Assuming steady, incompressible flow and no variation of pressure in z-direction and considering the centrifugal force acting in y-direction only, the Navier – Stokes
Figure 5: Surface forces in x and y-direction on an element of fluid.

The negative sign in Equation 1d indicates the decrease in pressure in y-direction. Now, considering the boundary conditions:

\[
\begin{align*}
\frac{\partial u}{\partial x} &= 0, \\
\frac{\partial v}{\partial x} &= 0, \\
\frac{\partial p}{\partial x} &= 0, \\
\frac{\partial^2 u}{\partial x^2} &= 0, \\
\frac{\partial^2 v}{\partial x^2} &= 0, \\
\frac{\partial^2 p}{\partial x^2} &= 0.
\end{align*}
\]

Where \( \omega \) and \( R \) are the angular velocity and radius of the wheel. Now, considering the circumferential symmetric flow around the rotating grinding wheel,

The equation is established as:

\[
\begin{align*}
\rho \left( \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} \right) &= - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\rho \left( \frac{\partial v}{\partial x} + \nu \frac{\partial v}{\partial y} \right) &= - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho \omega^2 R \\
\end{align*}
\]

And the continuity equation is given by:

\[
\left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) = 0
\]
At $y = 0$, $\frac{\partial p}{\partial y} = -\omega^2 R \rho$ and at $y = \delta$, $\frac{\partial p}{\partial y} = 0$.

A non-dimensional term is multiplied in Equation 1d. The index of expansion $n$, of the non-dimensional term $\left(1 - \frac{y}{\delta}\right)^n$ is taken as unity for the simplification of the theoretical development. As a result, the differential change in pressure in the air boundary can be obtained as:

$$dp = -\omega^2 R \rho \left(1 - \frac{y}{\delta}\right) dy$$

(2)

Where $\delta$ is the thickness of the boundary air layer. Integral of the aforementioned equation can give discrete values of pressure in the airfield and can be used to find the theoretical value of air pressure around the wheel in any point in $x$-direction as shown in Figure 5.

FORMULATION OF THE THEORETICAL MODEL WITH THE EXPERIMENTAL CONDITION

Equation 2 is a general expression of air pressure in the radial direction of a rotating grinding wheel and as such, this equation is required to be tested in the real-world case. Therefore, the parametric values at which the experimental investigations are carried out are applied in the theoretical model. The pressure of air is measured by the Pitot tube arrangement as shown in Figure 3 around a solid cast iron wheel and a grinding wheel. Therefore, two cases of modelling namely: Non-porous (solid) wheel and Porous (grinding) wheel are considered here.

Modelling for non-porous or solid wheel

Initially, experimentation was carried out on a non-porous wheel and the values of air pressure in radially outward direction noted. Therefore, in modelling this air layer formation, the value of pressure at the various radial distances of the non-porous wheel within the air boundary is required to be found out.

Theoretical model for non-porous wheel

Integrating the Equation 2 and considering the values of $\omega, R$ and $\rho$ as 2880 rpm, 200 mm and 1.225 kg/m$^3$ respectively as taken in the experimental observation, the value of $p$ can be written according to Equation 3 as:

$$p = -11131 \left(y - \frac{y^2}{2\delta}\right) + c$$

(3)

Considering boundary conditions:

$$p = 0 \text{ at } y = \delta$$

Equation 3 becomes:

$$p = -11131 \left(y - \frac{y^2}{2\delta}\right) + 142$$

(4)

In experimental observation, the Pitot tube is taken away radially from the wheel surface and positioned at 2 mm succession till no deflection of the water column is noted in the manometer. Thus, experimentally observed boundary layer thickness ($\delta$) is 25.5 mm. Now, applying this value of $\delta$ in Equation 4, the expression of air pressure obtained is shown in Equation 5:

$$p = -11131 y + 218254 y^2 + 142$$

(5)

Comparison of the theoretical model with the experimental values of non-porous wheel

At present, it is important to examine how good the theoretical model matches the experimentally observed data. In Figure 6, the theoretical values of air pressure obtained from Equation 5 are plotted along with the experimental values of air pressure around a rotating non-porous wheel. It displays a good match with the experimented values except for the region closure to the wheel periphery. The experimental values are noted by placing the Pitot tube in the periphery of the cast iron wheel where the axis of the tube is maintained at 3.5 mm distance from the wheel surface. The Pitot is placed at the centre of the wheel thickness initially and the position of it is changed at successive 2 mm distance from the initial 0-0 position to either side of it as shown in Figure 3. From each of these positions the Pitot tube is then moved radially outward (y-direction) at 2 mm succession. The results of the air pressure was noted from all 11 positions along the thickness (z-direction) of the wheel. The average of these results is considered to compare with the theoretical values of air pressure (Figure 6). After 7.5 mm radial distance from the wheel periphery, the theoretical curve matches better with the experimental values, but the significant difference between these two curves is observed near the wheel surface. At 3.5 and 5.5 mm radial distance from the wheel surface, a constant 33% increase in pressure is observed in experimental values than the theoretical, respectively. Thereafter, this difference diminishes quickly and after 7.5 mm both the
theoretical and experimental curves are found in a good match with each other. When a wheel rotates in the air, the particles of air closure to the wheel periphery are thrown outwards by the centrifugal action of the rotating wheel. The surrounding air from both the side faces of the wheel may then rush to counterbalance this outward flow of fluid (Figure 7). Even after compensating, more air continues to rush in the same direction due to their momentum and
collide near the mid-plane of wheel thickness (Majumdar et al., 2016). It is found to give a rise of the air pressure by 33% in the near vicinity of the wheel surface (Figure 6) which is not considered in the theoretical modelling. The consequence of this supplementary air due to which the pressure near the periphery of the solid wheel intensifies is termed ‘compensating air effect’. The effect of the compensating air is nearer the solid boundary and gradually diminishes within a short distance in the outward direction of the wheel. Less momentum energy of the compensating air may be the reason why the effect in the flow path was waned early.

Modification of theoretical model for Non-Porous Wheel

As the augmentation of air pressure due to the ‘compensating air’ is not considered in the present application of the Navier -Stokes equation, the theoretical model differs from the experimental. As a result of this compensating air, the experimental values of air pressure become more than the theoretical and observed closer to the wheel surface (Figure 6). When a cutting fluid is applied through a nozzle into the grinding zone, only 25% of the applied fluid can enter the contact surfaces. The air boundary layer formed around the wheel impedes the admission of this coolant (Morgan, 2008). Therefore, the study of the air flow field near to the wheel surface, which is more dynamic in obstructing the entry of the fluid jet is more important. Hence, a correction factor is required to be added to Equation 5 to compensate the deviation of the theoretical model from the experimental results at the near vicinity of the wheel periphery. Considering this effect, Equation 5 becomes:

\[ p = -11131 \gamma + 218254 \gamma^2 + 142 + (ae^{-by} + c) \]  

\[ (6) \]

Where \((ae^{-by} + c)\) is the correction factor added and \{a, b, c\} are arbitrary constants whose values are required to be calculated. It can be observed from Figure 5 that the theoretical graph decreases sharply initially. As the nature of the slope is found negative, the negative exponent is chosen in the correction factor. Further, Flower Pollination Algorithm (FPA) optimization technique has been employed for searching the best values of a, b and c such that the difference between the theoretical and experimental observation is optimally minimized.

Flower pollination algorithm to find out the parametric values of the correction factor

Flower Pollination Algorithm (FPA) is one of the recently proposed, efficient and popular SI based metaheuristic technique inspired by pollination of flowers (Yang, 2012; Yang et al., 2014).

Flower pollination is typically associated with the transfer of pollen for reproduction or flowering of plants and pollinators such as insects, birds and bats are mainly responsible for such transfer. FPA is based on some simplified rules for pollination. Biotic cross-pollination can be assumed as a process of global pollination and pollen carrying pollinators follow Lévy flights during transport (Rule 1). For local pollination, abiotic pollination and self-pollination are used (Rule 2). Pollinators may develop flower reliability, which is proportional to the resemblance of two flowers, that is, reproduction probability (Rule 3). The switching of local to global pollination can be controlled by a switch probability \(p \in [0, 1]\), slightly biased towards local pollination (Rule 4). Here, each pollen or flower corresponds to a solution of optimization problem. Global and local pollination (that is, search) are done according to the equations:

\[ x_i^{t+1} = x_i^t + \gamma \text{Lévy}(\lambda) (g_* - x_i^t) \]

\[ (7) \]

\[ x_i^{t+1} = x_i^t + \varepsilon (x_i^t - x_k^t) \]

\[ (8) \]

Where \(x_i^t\) is the pollen or solution vector \(x_i\) at iteration \(t\), \(\gamma\) is scaling factor to control the step, \(g_*\) is the current best solution found among all solutions at the current iteration, \(x_i^t\) and \(x_k^t\) are pollens from the different flowers of the same plant species and \(\varepsilon\) stands for random walk step size within a uniform distribution in \([0, 1]\), \text{Lévy}(\lambda) is Lévy flights distribution. The reason behind selecting FPA as optimization method is that it gives better convergence and accuracy than other popular metaheuristic techniques.

All optimization methods use an objective function or a fitness value to measure the goodness of a solution. The estimation task aims to seek the most optimal values for the unknown parameters so as to minimize the error between the measured and simulated pressure at the different location. The root mean square of the error (RMSE) is defined as Equation (9) can be used as the objective function (Mandal et al., 2017):

\[ RMSE(X) = \sum_{i=1}^{N} (f(y_i, X))^2 \]

Where \(N\) is the number of the experimental data, that is, a set of pressure and position, \(X\) is the set of the estimated parameters, that is, \(X = \{a, b, c\}\). For this present problem, \(f(V, d, l, X)\) can be given as:

\[ f(y_i, X) = p_{cal} - p_{exp} = 11131 \gamma + 218254 \gamma^2 + 142 + (ae^{-by} + c) - p_{exp} \]

\[ (10) \]

Where \(p_{cal}\) is the calculated air pressure using Eqn 6 for
Table 1: Statistical analysis of RMSE for solid wheel.

<table>
<thead>
<tr>
<th>RMSE</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>Median</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.959477</td>
<td>5.959477</td>
<td>5.959477</td>
<td>5.959477</td>
<td>7.83E-16</td>
</tr>
</tbody>
</table>

Table 2: Optimal values of a, b and c for solid wheel.

<table>
<thead>
<tr>
<th>Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>168.0499</td>
</tr>
<tr>
<td>b</td>
<td>0.4235</td>
</tr>
<tr>
<td>c</td>
<td>1.7496</td>
</tr>
</tbody>
</table>

a given set of \( y, a, b \) and \( c \). \( p_{exp} \) is the experimental air pressure that was obtained in laboratory. FPA tries to minimize the value of the RMSE error, such that best value of \( X = \{ a, b, c \} \) can be obtained.

In the present problem, dimension of search for the metaheuristic is 3 as the input variables of optimization process are \( \{ a, b, c \} \). For FPA, value of probability switch \( (p) \) is fixed to 0.8 using the guidelines provided by its own reference (Yang, 2012).

The number of population and total number of iteration are set as 50 and 1000 respectively. The lower and upper limit, that is, search range are chosen as \([-200, 200]\) respectively for \( \{ a, b, c \} \).

Initially, a total of 11 readings of pressure and distance were observed. Next, FPA was applied to this experimental dataset to find out the optimal values of \( a, b, c \). Experimental data is used for training data and calculation of the value of fitness function. After completion of iteration, FPA may reach the best solution, that is, the best combination of \( \{ a, b, c \} \) such that RMSE is minimized. At optimal condition, calculated \( P - y \) curve should be almost similar to experimental \( P - y \) curve for the grinding wheel. However, FPA may give different output depending on the initialization and randomness in search procedure. Therefore, FPA was executed 10 times. Thereafter, a statistical analysis was performed to obtain the final results.

It can be observed that for all of 10 individual runs, FPA is able to reach the optimal point where minimal value of RMSE is 5.959477. Table 1 shows the statistical analysis of the RMSE. It can be seen from the table that mean, maximum, minimum and median value of RMSE are almost same. Moreover, standard deviation is negligible which denotes that variation in fitness value is very small. Therefore, it can be concluded that the success rate of FPA for this optimization problem is very satisfactory. Table 2 shows the final optimal value of the \( a, b, c \) for solid wheel.

So, the modified theoretical model for the pressure \( (p_m) \) for solid wheel can be given as the:

\[
p_m = -11131y + 210254y^2 + 143.7496 + 168.0499e^{-0.4235y}
\]

(11)

Next, the values of pressures are calculated using Equation 11. Figure 8 shows a comparison among experimental pressure values with the simulated output. It can be observed from Figure 8 that there is very little difference between experimental and simulated \( P - y \) characteristics. This statement validates our proposed methodology to establish the theoretical modelling of air flow field around the solid wheel.

Modelling for porous or grinding wheel

The modified theoretical model (Equation 11) is good enough for finding the pressure around a rotating non-porous wheel, but whether this model can be consistent with the air flow field of the grinding wheel also is required to be tested. A grinding wheel is porous and has the surface roughness. Therefore, the pressure of the boundary air layer is more around the grinding wheel than the solid wheel (Majumdar et al., 2016).

The flow of air also may take place through the wheel in the outward direction and roughness of the wheel surface also increases the air boundary layer pressure (Majumdar et al., 2016).

The outward flow of air through the porous wheel and the compensating air flow together is termed as spouting (Marinescu et al., 2012). Hence, in modelling the air pressure around a grinding wheel, the increase in air pressure due to spouting and surface roughness are also considered.

To include these two effects in the theoretical model, Equation 6 is utilized and the term \( (ae^{-by} + c) \) is
Figure 8: Comparison among experimental pressure values with the simulated output for non-porous wheel.

Table 3: Statistical analysis of RMSE for solid wheel.

<table>
<thead>
<tr>
<th>RMSE</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>Median</th>
<th>Standard deviation</th>
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<tbody>
<tr>
<td></td>
<td>5.295331</td>
<td>5.295331</td>
<td>5.295331</td>
<td>5.295331</td>
<td>2.74E-15</td>
</tr>
</tbody>
</table>

Figure 9: Comparison among experimental pressure values with the simulated output for non-porous wheel.

optimized again for finding the suitable value of a, b and c using FPA. FPA was implemented with same operating conditions and settings. The optimum values of the parameters a, b and c (for the grinding wheel) are given in Table 3. In this case, mean of RMSE is 5.295331 and standard deviation is very small indicating the stability of the FPA optimization. Thus, the modified model of the air boundary layer pressure around the grinding wheel becomes:

\[ p_m = -11131y + 218254y^2 + 151.2359 + 457.8499e^{-0.2374y} \]  \hspace{1cm} (12)

Figure 9 shows the experimental and the model of the air flow around the grinding wheel. It is found that the model
curve matches very well with that of the experimental and the model is consistent with the results of the experimental.

CONCLUSION

The pressure field generated due to the rotation of a non-porous wheel does not only depend upon the surface and centrifugal forces on air particles but also the effect of compensating air which rushes from both sides of the wheel towards the periphery. Its effect is found to dominate the flow field at the near vicinity of the wheel periphery. In this experiment, its effect is observed to be dominant up to 5.5 mm distance from the wheel surface and which diminishes at the distance of 7.5 mm from the wheel periphery. A nearly 33% increase of pressure is observed due to the compensating air effect.

In the case of grinding wheel, the effect of surface roughness of the wheel and the spouting are responsible for the additional growth of air pressure. The novel correction factor added with the theoretical model is found good enough to match the developed modified model with the experimentally observed data in both non-porous and porous wheel. FPA was found good for optimizing the parameters of the correction factor.

The proposed model can be utilized in predicting the air pressure in the tested flow field without any physical experiment. The testing of the generated model with the wheels of different porosities and grain sizes is the future scope of this work.

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