Mono-Energetic Proton Radiography Due to Laser Fusion in Magnetic Field Determination

Accepted 14th, September 2020

ABSTRACT

Mono-energetic proton radiography is a useful diagnosis of high-energy density plasmas that is expanding in both theoretical and experimental domains. Scheduled mono-energetic proton radiography presents unique measurements of electric and magnetic fields regarding to laser-foil interactions, exploded capsules by inertial confinement fusion and laser irradiated hohlraums. These experiments led to the first observations of several new and important features that were not previously addressed. Such as generation observations, decay dynamics, instabilities and a change in the geometry of mega-gauss B fields on laser driven flat plastic plate foil. In this paper at first we determine the fusion energy of D-3He reaction in order to be used in industry and then we use of mono-energetic 3 and 14.7 MeV generated protons from D-3He fusion reactions for proton radiography and then the physics of the produced magnetic field will be discussed for produced protons of D-3He fusion reactions with energy of 3 and 14.7 MeV.

Key words: Magnetic field, proton, mono-energetic, laser, energy.

INTRODUCTION

In the mid-1990’s, proton radiography was invented at Los Alamos (Gavron et al., 1996). The proton radiography uses the attenuation because of the nuclear scattering of very short pulses of energetic protons as they transit high explosive driven experiments to provide contrast for flash radiography. Proton radiography uses attenuation due to nuclear scattering of very short pulses of energetic protons when they pass through the matter. The interaction of high-energy protons with matter largely involves nuclear and Coulomb interactions. Protons lose their energy to the matter due to the Coulomb scattering of the protons from the electrons of atoms and their scattering from the nuclei. The proton energy loss is discussed by the Bethe-Bloch equation (Bethe, 1930), Coulomb scattering is determined by the Moliere equation (Bethe, 1953; Moliere, 1948), and nuclear scattering can be given by optical model of nuclear scattering (Henley and Garcia, 2007). Proton radiography is highly challenging in terms of dose delivery, imaging speed, image quality (compression and spatial resolution), and image content under clinical conditions. Protons with approximate energy of 250 MeV can penetrate in the patient's body and take imaging. Proton radiography is a new technology for the interior imaging of objects that is obtained by penetrating of energetic protons. A wide beam of energetic protons shines through the objects and they are attenuated by their collisions with the nuclei. Also, the number of protons in their path is reduced due to multiple Coulomb scattering (MCS). The produced protons from D-3He fusion reaction move from the source to the interaction zone containing the high energy density plasma, scatter from the electromagnetic fields due to the Lorentz force, then propagate to a distant detector where the radiograph is recorded.

A magnetic lens, after extracting an implicit image, again focuses on the detector with a different scattering angle, and there it records them with the help of television cameras. The recognition of electromagnetic field generation driven by intense laser-matter interactions is especially important for high energy density plasma physics. This method for radiography created uniquely high performance diagnostic, imaging high energy density plasmas with increasing the spatial resolution of several micrometers and range of temporal resolution 1 ≤ t ≤ 10ps. Studies show that the proton radiography diagnostic method has a remarkable success by creating background of electromagnetic fields recognition in the mega-gauss scale for imploding using inertial confinement fusion (ICF) (The National Academies Press, 2003; Zylstra et al., 2012),

S. N. Hosseinimotlagh
Department of Physics, Azad Islamic University, Shiraz Branch, Iran.
E-mail: hosseinimotlagh@hotmail.com
self-organized electromagnetic field structures of large scale in terms of large speed counter streaming plasma flows (Kugland et al., 2012), magnetic reconnection process (Nilson et al., 2006; Willingale et al., 2010), high energy density plasma instabilities (Li et al., 2007; Gao et al., 2013), and so on (Hogan et al., 1999; Borghesi et al., 2008). Since the aim of this work is investigating the mono-energetic proton radiography due to laser fusion in magnetic field determination, therefore the study is organized as follows: discussion on the balance equations of D+3He fusion reaction in order to determine the particles, energy and proton flux generated using in radiography are presented; proton radiography mechanism in determining the fields created by laser-plasma interactions is introduced; investigating the physical theory of proton radiography in order to determine magnetic field and proton intensity on the imaging plane. Finally, we give a brief summary of our work.

**BALANCE EQUATIONS OF D+3HE FUSION REACTION**

In the last few decades, the use of nuclear fusion energy using inertial confinement method has received much attention and various designs have been proposed to achieve high energy efficiency. In conventional methods, the existence of hydrodynamic instabilities reduces the energy gain. Another method which is called "fast ignition" has been proposed to reduce instability and obtain high energy gain. In this method, unlike the central hot spot ignition method, compression and ignition processes are separated. So far, there are different driven beams such as relativistic electrons and beams of charged particles such as light and heavy ions have been used for ignition. Each of them has different advantages and disadvantages. In light ions, deuteron beams in addition to heating are used as fuel of fusion reactors and can generate high energy densities by stopping in smaller volumes of fuel. Therefore, obtaining additional fusion energy gain with them is a unique feature that can reduce the total required deuterons flux or alternatively reduce the energy of the incident laser beam. The purpose of this section is to carry out investigation on the fast ignition of D-3He fuels using deuteron beams and calculate their added energy gain. The following system of equations is used to describe the temporal evolution of plasma parameters averaged over the volume (the density of deuterium ions nD, density of helium-3 ions n3He, density of thermal alpha-particles nα, density of protons p, plasma energy E), for D+3He nuclear fusion reaction:

\[ \frac{dn_0}{dt} = -\frac{\Sigma n_0}{\tau_p} - n_D n_{3He} \langle \sigma v \rangle_{D+3He} + S_D \]  

\[ \frac{dn_{3He}}{dt} = -\frac{\Sigma n_{3He}}{\tau_p} - n_D n_{3He} \langle \sigma v \rangle_{D+3He} + S_{3He} \]

The energy balance is given by:

\[ \frac{dE}{dt} = -L + Q_{\alpha} n_D n_{3He} \langle \sigma v \rangle_{D+3He} - P_{\text{rad}} \]  

Where \( A_b = 5.35 \times 10^{-37} \frac{\text{Wm}^3}{\text{s}^2} \) is the bremsstrahlung radiation coefficient. No explicit evolution equation is provided for the electron density ne since we can obtain it from the neutrality condition \( n_e = n_D + n_{3He} + 2n_\alpha \), whereas the effective atomic number, the total density and the energy are written as:

\[ Z_{\text{eff}} = \frac{\Sigma nZ_i^2}{n_e} = \frac{n_D + n_{3He} + 4n_\alpha}{n_e} \]

Where \( Z_i \) is the atomic number of the different ions. Also, the flux of protons generated by the fusion reaction \( ^3\text{He} + D \rightarrow P(14 \cdot 7\text{MeV}) + ^3\text{He}(3 \cdot 6\text{MeV}) \) is obtained by \( \Phi_p = n_p v_p \), where \( n_p \) and \( v_p \) are the density and velocity of the protons produced by D-3He reaction, respectively. Notice that

\[ v_p = \sqrt{\frac{2E_p}{m_p}} \]

Where \( E_p \) and \( m_p \) are kinetic energy and mass of proton. The fusion energy gain is defined as:

\[ G(t) = \frac{E(t)}{E_{\text{driver}}} \]

Where \( E(t) \) is equal to the energy due to the number of occurred fusion reactions in target in terms of time and \( E_{\text{driver}} \) is the required energy for triggering fusion reactions in hot spot and is equal to 4MJ. Also the fusion power density for D+3He reaction is given by:

\[ P_{D+3He} = n_D(t)n_{3He}(t) \langle \sigma v \rangle_{D+3He} Q_{D+3He} \]

Where \( Q_{D+3He} = 18.3\text{MeV} \).
We solve Equations (1), (2), (3), (4) and (5) in dynamical state (time-dependent density of atoms) with the use of computers (programming, Maple-18) under available physical conditions. Our computational obtained results are given in Figure 1. From this figure we see clearly that by increasing temperature from 1 keV to 180 keV the...
variations of deuterium and helium-3 density in terms of 
time \( (n_D(t), n_{3He}(t)) \) are decreased since by increasing time, 
the consumption rate of \( n_D(t) \) and \( n_{3He}(t) \) are increased. 
While, by increasing temperature 1keV to 180 keV the 
variations of alpha and proton densities (or proton flux) 
\( (n_\alpha(t), n_p(t)) \)or \( \Phi_p(t) \) versus time, similarly at first by 
increasing time are increased and then decreased. Notice 
that this proton flux can be used to proton radiography. Also the rate production of fusion plasma 
energy \( (E(t)) \) at first is increased and finally reach to 
constant value, but with increasing the temperature this 
energy is increased. It is important that the D-3He fusion 
reaction has two advantages. Firstly we can use of 
generated fusion energy in industry and secondly by 
filtering and reduction the proton flux it can be used in 
proton radiography.

PROTON RADIOGRAPHY MECHANISM IN DETERMINING 
THE FIELDS CREATED BY LASER- PLASMA INTERACTIONS

Backlighters are used in simple radiographic setup as 
shown in Figure 2, but their isotropic properties cause at 
least 2 different cases at various angles respect to the 
backlighter, light up to maximize the data due to the 
certain number of photos. Proton flux images are captured 
by the CR-39 with automated scanning systems. The 
incident energy of each individual proton is also 
determined by its path in the CR-39. After the NaOH 
detectors were invented, the radiation positions of each 
individual proton were identified and measured on a nano 
scale (Kugland et al., 2012; Ryutov et al., 2013; Kugland 
et al., 2013). The single-energy nature of the particles used for 
imaging enables us to perform two types of measurements 
simultaneously for the imaging cases. In the first type, 
particle energy indicates how much energy a particle has 
lost as it passes through the material, thus from it we can 
determine the surface density of the material. In the second 
type, the deviations created in the particle trajectory by the 
E and B fields can be measured to quantify the field 
intensity. In order to quantitatively investigate laser and 
plasma interactions, particularly the resultant 
electromagnetic fields an imaging system that connects a 
mono-energetic particle backlighter with a detection-
matched system is provided. This approach has advantages 
over radiography with proton sources of broadband. The 
single-energy particles (such as 3.6 MeV alphas, 3 MeV 
and 14.7 MeV protons) are nuclear fusion products, which 
generated by laser fusion of D3He and D-D reaction. 
Note that the fuel capsule is filled with deuterium and 
he helium fuel, in which D-3He fusion is main reaction while D-
D is side reaction and are given by:

\[
\begin{align*}
D + D & \rightarrow P(3\text{MeV}) + T(1\text{MeV}) \quad (11) \\
D + ^3\text{He} & \rightarrow \alpha(3.6 \text{ MeV}) + p(14.7\text{MeV}) \quad (12)
\end{align*}
\]

Implosions of backlighter are generally excited by 20 or less 
OMEGA laser beams (0.351 µm wave length) with a square 
pulse of 8-10 kJ (2.5 to 5 kJ), with pulse duration 1-ns (0.6 
ns). The capsule diameter is small and about 440 µm to 
provide a lower radius than normal to improve the spatial 
resolution of the radiograph. A complete set of diagnostic 
functions were used to characterize implosion features, 
including proton and X-ray emission images to study the 
size of the imploded capsule and its burn zone, and a proton 
time diagnostic test to measure the burn time. Each fusion 
product is the single-energy and produced just in during 
130 picoseconds.

THE PHYSICAL THEORY OF PROTON RADIOGRAPHY

The analytical theory of proton radiography describes the 
relationship between electromagnetic fields and the flux 
images generated by protons in the sample. Imagine a 
Gaussian ellipsoidal halo containing magnetic field with 
only an azimuthal component (\( \varphi \)):

\[
B_{\varphi} = B_0 \frac{r}{a} \exp \left( -\frac{r^2}{a^2} - \frac{z^2}{b^2} \right)
\]
Figure 3: The a) 3 and b) 2-dimensional variations of $\frac{B_\phi}{B_0}$ as a function of $b$ and $z$.

Figure 4: The a) two and b) three dimensional variations of $\alpha$ in terms of different values $B_0$ and $b$.

Where $r$ is radial coordinate, $z$ is axial coordinate, the parameters $a$ and $b$ are called semi-major and major axes, respectively.

For the case of $b > a$, the structure of this field is like a single Weibel’s instability driven magnetic filament (Kugland et al., 2012; Ryutov et al., 2013; Kugland et al., 2013). Remember that $B_0$ is not the maximum value of the $B$ field. The $B$ field maximum value is obtained at $r = a\sqrt{2}$, and is equal to: $B_{\text{peak}} = B_0/\sqrt{2e} \approx 0.43 B_0$. The distance from the source to the center of the object is $|z_s| = 1$ cm and the distance from the center to the plane of image is $z_i = 10$ cm, the energy of proton is obtained from $\epsilon_p = \frac{1}{2}m_pv_p^2 = 14.7$ MeV, for the proton with mass $m_p$ and velocity $v_p$. We consider $a = 100$ μm and $b = 300$ μm. Therefore, this mode is compatible with the approximation of $a/|z_s| \sim 10^{-2}$ (see Figure 3). It is seen that from Figure 3, the ratio $\frac{B_\phi}{B_0}$ increases with increasing $b$ but decreases with increasing $z$. In the analytical study of the proton deflection, we use the $b$ dimension smaller than the gyroscope radius $\rho \sim 3$ cm for sharp fields. This allows us to use a linear approximation. Linear approximation is an integration on the transverse force over the non-continues direct path inside the structure of the field. The suggested error of this assumption is lower than 10%. With that, we conclude that the angle of deflection $\alpha$ is proportional to radius $r_0$ that corresponds to the point at which the protons collide with the surface of the object so that:

$$\alpha = \mu \frac{r_0}{a} \exp \left( -\frac{r_0^2}{a^2} \right)$$

(14)

Where

$$\mu = \frac{\sqrt{\pi} |e| B_0 b}{m_p v_p c}$$

(15)

$\mu$ is a dimensionless parameter in which corresponding to the interaction and $e$ is the elementary charge. For the proton source with 14.7 MeV energy, we have $\frac{v_p}{c} = 0.177$ and $\mu = 3.2 \times 10^{-6} B_0 [T] b [\mu m]$. In order to understand the variations of $\mu$ and $\alpha$ in Figures 4 and 5 we have plotted the two and three dimensional variations of $\alpha$ and $\mu$ in terms of parameters $B_0$ and $b$. It can be seen that from Figures 4 and 5, with increasing $B_0$ and $b$, the quantities $\mu$ and $\alpha$ are increased. The position of the point on the image
Figure 5: The a) two and b) three dimensional variations $\mu$ in terms of different values $B_0$ and $b$.

Figure 6: The a) two and b) three dimensional variations of $r = z_i \left( -\frac{r_0}{z_s} - \alpha(r_0) \right)$ in terms of different values $B_0$ and $b$.

Figure 7: The a) two and b) three dimensional variations of $r = z_i \left( -\frac{r_0}{z_s} + \alpha(r_0) \right)$ in terms of different values $B_0$ and $b$.

The plane is determined by the following equation:

$$r = z_i \left( -\frac{r_0}{z_s} \pm \alpha(r_0) \right)$$  \hspace{1cm} (16)

Where the negative and positive sign are related to the focusing and non-focusing states respectively (Figures 6 and 7). It can be seen that from Figure 6, with increasing $B_0$ and $b$, the absolute value of $r$ decreases, while from Figure 7
we found that with increasing $B_0$ and $b$ the absolute value of $r$ is increased. $\frac{dr}{dr_0}$ is given by:

$$\frac{dr}{dr_0} = \frac{z_i}{z_s} \left[ 1 \pm \frac{\mu|z_s|}{a} f\left(\frac{r_0}{a}\right) \right]$$

(17)

In order to see the behavior of $\frac{dr}{dr_0}$ variation for negative and positive signs, we have the two cases (see Figures 8 and 9). From these Figures, we find that by considering the negative (positive) sign of equation (17) the value of $\frac{dr}{dr_0}$ increases (decreases) with increasing $b$ and $B_0$.

For a small value of $\mu$ (small magnetic field) the second term can be ignored, and only the first term has a uniform magnification. When $\mu$ is increased, the condition: $\frac{dr}{dr_0} = 0$ eventually reaches to some of the different $\mu_{\text{crit}}$ values for focusing and non-focusing modes. For a focusing mode, this critical value is equal to:

$$\mu_{\text{crit}} = -\frac{a}{z_0}$$

(19)

While for a defocusing mode, this critical value is equal to:

$$\mu_{\text{crit}} = -\frac{1}{2} \frac{z_0}{a^2} \approx -2.24 \frac{a}{z_0}$$

(20)

For viewing the variations of $\mu_{\text{crit}}$ in terms of $z_0$ related to two focusing and defocusing modes (Figure 10).

By introducing the values of the global constants, the following expressions are obtained for critical magnetic field:

$$B_0_{\text{crit}}[T] = -8.12 \frac{a}{b} \frac{[\text{MeV}]}{z_0[\text{cm}]}$$

(21)
The above relations are related to critical magnetic fields in the focusing and defocusing modes, respectively. Using the input parameters for these sample states, we obtain fields with values 10.38 and 23.26 T, respectively (See Figure 11). Figures 11a and b show variations of $B_{0\text{crit}}$ for the protons from the fusion reactions (11) and (12), which have a kinetic energy of 14.7 and 3 MeV, respectively. As you can see, for focusing and defocusing modes with increasing $b$ parameter for both of $\varepsilon_p = 14.7$ MeV and $\varepsilon_p = 3$ MeV the value of $B_{0\text{crit}}$ is decreased. But the value of $B_{0\text{crit}}$ for $\varepsilon_p = 14.7$ MeV is larger than $\varepsilon_p = 14.7$ MeV for each case of focusing and defocusing. Using equations (14) to (18), the distribution of intensity on the image plane for $\mu$ less than the critical value can be presented in terms of a parameter $t$ as follows:

$$I = \left|e^{-2t^2} \left( v \mp v^2 e^{-2t^2} \right) \left( e^{t^2} \mp v(1 - 2t^2) \right) \right|^{-1}$$

(23)

$$R = \frac{1}{\hat{R}} \left|1 \mp ve^{-t^2} \right|, \quad v \equiv -\frac{\mu zs}{a}$$

(24)

Where, $I_0$ is the intensity at the center of the image plane in the absence of the object, and $R$ is equal to: $R = -\frac{z_i a}{z_s}$. Figure 12 shows the two-dimensional variations of $v$ in terms of $b$ and $B_0$ parameters. As can be seen, the absolute value of $v$
increases with increasing $B_0$ and $b$. Also, in Figure 13, show that the variations of $v_{crit}$ using $\mu_{crit}$ for focusing and defocusing cases in terms of $Z_0$. In Figures 14 and 15, the distribution of the intensity related to of equation 23 are given on the image plane for $\mu$ less than the critical value in terms of the $t$ parameter for three different values of $b = 300 \mu m$, $b = 400 \mu m$ and $b = 500 \mu m$ for $\frac{r}{R} = t \{1 \pm ve^{-t^2}\}$ at the range of $0 \leq \frac{r}{R} \leq 3$. As we can see in Figures 14a and d, the yellow and blue colors are devoted to $+$ sign of equation 23 for $b = 300 \mu m, 0 \leq \frac{r}{R} \leq 3$ and $\frac{r}{R} = t \{1 \pm ve^{-t^2}\}$. From these figures, it is understood that the highest intensity

- Figure 12: The two-dimensional variations of $v$ in terms of $B_0$ and $b$.

- Figure 13: The two-dimensional variations of $v_{crit}$ in terms of $Z_0$ for two cases of focusing and defocusing.
Figure 14: Distribution of intensity on the image plane for $\mu$ less than the critical value in terms of the $t$ parameter and the various values of $a$: b: $300\,\mu m$, c: $400\,\mu m$, d: $500\,\mu m$ and $b: b = 300\,\mu m$, e: $b = 400\,\mu m$, and f: $b = 500\,\mu m$ for $r = \frac{t}{1 - v e^{-t}}$. [$1 + v e^{-t}]$ related to two modes: 1: $I_0 = |e^{-2t^2}(v - e^{t^2})(e^{t^2} - v(1 - 2t^2))|^{-1}$ (green and red) and 2: $I_0 = |e^{-2t^2}(v + e^{t^2})(e^{t^2} + v(1 - 2t^2))|^{-1}$ (yellow and blue).

distribution of $I_0$ is slightly more than 1.02 for $t = 0$, which gradually decreases with increasing $t$ and reaches a minimum approximate value of 0.995 at $t = 1.5$. Then, by increasing the magnitude of $t$, the value of $\frac{I}{I_0}$ increases and at $t = 3$ approaches to approximately value of 1, and similarly at the Figures 14a and d for selecting the $- -$ sign of $a$. 

equation 23 we have the green and red diagrams, which are the opposite of the ++ sign, that is, by increasing \( t \), the value of \( \frac{I}{I_0} \) from zero gradually increases and reaches a maximum value of 1.005 at \( t = 1.5 \), and then by increasing the value of \( t \) its value decreases and at \( t = 3 \) approaches an approximate value of 1. In Figures 16 b and e the graphs with yellow and blue colors are devoted to the equation 23 with considering the ++ sign for \( b = 400 \mu m, 0 \leq \frac{r}{R} \leq 3 \) and \( \frac{r}{R} = t \right| 1 \pm \text{e}^{-t^2} \). From these Figures we find that, the highest intensity distribution of \( \frac{I}{I_0} \) is slightly more than 1.06 at \( t = 0 \), which gradually decreases with increasing \( t \) and reaches a minimum value of 0.99 at \( t = 1.4 \) and then it increases with increasing \( t \) and approaches the value of 1 at \( t = 3 \).

Also in Figures 16, b and e for the -- sign, we have the green and red diagrams using equation 13, which has the opposite behavior of the ++ sign, that is, with increasing \( t \), the value of \( \frac{I}{I_0} \) gradually increases from zero value, and

\[
\frac{I}{I_0} = |e^{-2t^2}(v - e^{t^2}) (e^{t^2} + v(1 - 2t^2))|^{-1} \text{ (yellow and blue).}
\]

\[
\frac{I}{I_0} = |e^{-2t^2}(v + e^{t^2}) (e^{t^2} - v(1 - 2t^2))|^{-1} \text{ (green and red).}
\]
Figure 16: the distribution of intensity on the image plane for fields greater than the critical values in terms of the t parameter for $\varepsilon = 0.5$ and the different values of $a$: $b = 300\mu m$, $b = 400\mu m$, $c: b = 500\mu m$ for $\frac{R}{R} = t[1 - ve^{-t^2}]$ and $d: b = 300\mu m$, $e: b = 400\mu m$, and $f: b = 500\mu m$ for $\frac{R}{R} = t[1 + ve^{-t^2}]$ related to two modes: 1: $I = I_0 = \frac{1}{1 + ve^{-t^2}(v - e^{t^2})[e^{t^2} - v(1 - 2it^2)]}$ (blue and orange) and 2: $I = I_0 = \frac{1}{1 + ve^{-t^2}(v + e^{t^2})[e^{t^2} + v(1 - 2it^2)]}$ (violet and red).

reaches the maximum approximate value of 1.01 at $t = 1.4$ and then decreases with increasing the value of $t$ and approaches the approximate value of 1 at $t = 3$. We find that from Figures 14c and f (yellow and blue diagrams) when we consider the equation 23 are devoted to the ++ sign for $b = 500\mu m, 0 \leq \frac{R}{R} \leq 3$ and $\frac{R}{R} = t[1 - ve^{-t^2}]$.

These figures show that the highest intensity distribution of $\frac{1}{I_0}$ is slightly more than 1.10 at $t = 0$, which gradually decreases with increasing $t$ and reaches a minimum value of 0.98 at $t = 1.4$, and then it increases with increasing $t$ and approaches to value of 1 at $t = 3$, also Figures 14 c and f (green and red diagrams) are devoted to - - sign in the equation of 13 in which has the opposite behavior of the ++ sign, that is, with increasing $t$, the value of $\frac{1}{I_0}$ is gradually increased from zero and reaches a maximum value of 1.01.
at \( t = 1.5 \), and then decreases with increasing value of \( t \) and approaches an approximate value of \( 1 \) at \( t = 3 \). Finally, from the diagrams of a and d, b and e, c and f, we conclude that all diagrams have the same behavior but with increasing \( b \) from 300 to 500 \( \mu m \) gradually, the value of \( \frac{1}{\rho_0} \) is slowly increasing, as well as the mentioned graphs for \( \frac{I}{\rho} = t \left| 1 \pm ve^{-t^2} \right| \) in two different ways, it's positive and negative sign does not make a large change in the value of \( \frac{1}{\rho_0} \). Similar to Figure 14, in Figure 15 we plotted, the distribution diagram of the intensity \( \frac{1}{\rho_0} \) related to equation 23 on the image plane for \( \mu < \) the critical value in terms of the \( t \) parameter for three different values \( b = 300 \mu m, b = 400 \mu m \) and \( b = 500 \mu m \) for \( \frac{r}{\rho} = t \left| 1 \pm ve^{-t^2} \right| \) and \( 0 \leq \frac{r}{\rho} \leq 3 \). In Figures 15a and d, (blue and yellow diagrams) for selecting + sign of equation 23, we find that for \( b = 300 \mu m, 0 \leq \frac{r}{\rho} \leq 3 \) and \( \frac{r}{\rho} = t \left| 1 \pm ve^{-t^2} \right| \) at \( t = 0 \), the value of \( \frac{1}{\rho_0} \) is equal to 1 and increases gradually with increasing \( t \), so that at \( t = 1 \) it reaches a maximum approximate value of 1.075, and then with increasing \( t \) its value gradually decreases and at \( t = 3 \) is approached to 1. Also in Figures 15a and d (green and red diagrams) for selecting + sign in equation 23, which has the opposite behavior of the + sign, that is, at \( t = 0 \), the value of \( \frac{1}{\rho_0} \) is equal to 1 and gradually with an increase in \( t \) parameter the value of \( \frac{1}{\rho_0} \) decreases and reaches an approximate value of 0.888 at \( t = 1 \). Then, by increasing the value of \( t \), the value of \( \frac{1}{\rho_0} \) gradually increases and at \( t = 3 \) they approach the unit value. From Figures 15b and e (blue and yellow diagrams), we find that for selecting the - + sign of equation 13, for \( b = 400 \mu m, 0 \leq \frac{r}{\rho} \leq 3 \) and \( \frac{r}{\rho} = t \left| 1 \pm ve^{-t^2} \right| \) at \( t = 0 \), the value of \( \frac{1}{\rho_0} \) is equal to 1 and increases gradually with increasing \( t \), and at \( t = 1 \) it reaches an approximate value of 0.025, and then with increasing \( t \) its value gradually decreases and at \( t = 3 \) its value is close to 1. In Figures 15b and e (green and red diagrams) for selecting + - sign in equation 23, we see opposite behavior of the + sign that is, at \( t = 0 \) the value of \( \frac{1}{\rho_0} \) is equal to 1, and gradually, with increasing \( t \), the value of \( \frac{1}{\rho_0} \) decreases and reaches an approximate value of 0.875 at \( t = 1 \).

Then, by increasing the value of \( t \), the value of \( \frac{1}{\rho_0} \) gradually increases and at \( t = 3 \) they approach the unit value. From Figures 15c and f (blue and yellow diagrams) for selecting + - sign in equation 23, for \( b = 500 \mu m, 0 \leq \frac{r}{\rho} \leq 3 \) and \( \frac{r}{\rho} = t \left| 1 \pm ve^{-t^2} \right| \) at \( t = 0 \), the value of \( \frac{1}{\rho_0} \) is equal to 1 and increases gradually with increasing \( t \), so that at \( t = 1 \) it reaches a maximum approximate value of 1.041, and then with increasing \( t \) its value gradually decreases and reaches to approximately value 1 at \( t = 3 \). And in Figures 15c and f (green and red diagrams) can be seen that for selecting + - sign in equation 23, the opposite behavior of the + - sign, that is, at \( t = 0 \), the value of \( \frac{1}{\rho_0} \) is equal to 1 and gradually decreases with increasing \( t \) and it reaches an approximate value of 0.8575 at \( t = 1 \). Then, by increasing the value of \( t \), the value of \( \frac{1}{\rho_0} \) gradually increases and at \( t = 3 \) they approach the unit value. Note that in plotting the Figures 15a, b and c we use: \( \frac{r}{\rho} = t \left| 1 \pm ve^{-t^2} \right| \), while in the graphs 15d, e and f we use: \( \frac{r}{\rho} = t \left| 1 + ve^{-t^2} \right| \) in which in these Figures it can be seen that almost have identical effects. Also, in general, the yellow and blue graphs behave similar to each other and vice versa, the green and red diagrams behave similarly, but with increasing \( b \) the value of \( \frac{1}{\rho_0} \) is slightly increases. It is also possible to draw intensity distributions for fields with more than critical values. Therefore, in order to do this, domain constraint factor \( \varepsilon \) must be given in equation 23 (Kugland et al., 2013). Then, a suitable parametric relation for the intensity of the normalized image plane in consistent with equation 24 for \( r = \rho \) is given by:

\[
\frac{1}{\rho_0} = \frac{1+\varepsilon}{t+|e^{-2\pi^2(\nu - ve^{-t^2})}(\varepsilon^2 + \varepsilon^2(1-2t^2))|}
\]

In Figure 16, we have plotted the distribution diagram of the intensity of the normalized image plane corresponds to equation 25 for fields with more than critical values in terms of the \( t \) parameter using three values \( b = 300 \mu m, b = 400 \mu m \) and \( b = 500 \mu m \) for \( \frac{r}{\rho} = t \left| 1 \pm ve^{-t^2} \right| \) and \( 0 \leq \frac{r}{\rho} \leq 3 \). In Figures 16a and d, we have plotted the intensity distribution diagrams corresponding to the equation 25 on the image plane for fields with more than critical values in terms of the \( t \) parameter using three different values \( b = 300 \mu m, b = 400 \mu m, b = 500 \mu m \) for \( \frac{r}{\rho} = t \left| 1 \mp ve^{-t^2} \right| \), and \( 0 \leq \frac{r}{\rho} \leq 3 \) and \( \varepsilon = 0 \cdot 5 \). From these Figures, it is understood that when we take ++ sign in equation 25, for \( b = 300 \mu m \) (violet and red diagrams), the highest intensity distribution of \( \frac{1}{\rho_0} \) is approached to the approximated value of 1.002 for \( t = 0 \), which gradually decreases with increasing \( t \) and reaches a minimum value at 0.9995 at \( t = 1.4 \) and then increases with increasing \( t \) and at \( t = 3 \) reaches to the approximately value of 1.

Similarly, in Figures 16a and d, for-- the sign in equation 25 (blue and orange diagrams), we find that they have the opposite behavior of ++ sign, that is, as \( t \) increases, the value of \( \frac{1}{\rho_0} \) gradually increases from zero and reaches to a maximum value 1.0005 at \( t = 1.4 \). Then decreases with increasing value of \( t \) and approach to an approximate value 1 at \( t = 3 \). Similarly, it is found from Figures 16b and e when we consider ++ sign in the equation 25 for \( b = 400 \mu m, 0 \leq \frac{r}{\rho} \leq 3 \) and \( \frac{r}{\rho} = t \left| 1 \pm ve^{-t^2} \right| \) (violet and red diagrams) at \( t = 0 \) the intensity distribution value of \( \frac{1}{\rho_0} \) is zero, as \( t \) increases
gradually its value increases and at \( t = 0.6 \) approaches to the approximately value 0.5. And then decreases with \( t \) and increases again at \( t = 1.8 \), so that at \( t = 2.2 \) the value of the violet and red graphs are respectively 2.9 and 8.8, 2, then with increasing \( t \) to \( t = 2.4 \), the value of \( \frac{1}{I_0} \) (red and purple diagrams) reach to the approximately values 0.5 and 0.4, respectively, then with increasing \( t \) its value increases and at \( t = 3 \) the value of \( \frac{1}{I_0} \) (violet and red diagrams) will reach to the approximately values 0.8 and 0.9, respectively.

It is understood from Figure 16b and e, when we take the sign in equation 15, for \( b = 400 \mu m, 0 \leq \frac{r}{R} \leq 3\text{and} \frac{2}{R} = t\left[1 \pm ve^{-t^2}\right] \) (blue and orange diagrams) at \( t = 0 \), the amount of intensity distribution of \( \frac{1}{I_0} \) for each of the two colors is zero. As \( t \) increases gradually, the values of \( \frac{1}{I_0} \) corresponds to the blue and orange diagrams at \( t = 0.6 \) reach to the approximately values 0.3 and 0.2, respectively. Then, with increasing \( t \), the value of \( \frac{1}{I_0} \) decreases and at \( t = 1.8 \) again increases, at \( t = 2.7 \) the value of \( \frac{1}{I_0} \) for (blue and orange diagrams) reaches to 2.95 and 2.9, respectively, and then decreases with \( t \) increasing, at \( t = 3 \) the values of \( \frac{1}{I_0} \) (blue and orange diagrams) reach approximately values 1.2 and 1.25, respectively. It is understood that from Figure 16c and f when we take \( + \) or \( - \) sign in equation 15, for \( b = 500 \mu m, 0 \leq \frac{r}{R} \leq 3\text{and} \frac{2}{R} = t\left[1 \pm ve^{-t^2}\right] \) (violet and red diagrams) at \( t = 0 \) the amount of intensity distribution of \( \frac{1}{I_0} \) is zero, as \( t \) increases gradually, the value of \( \frac{1}{I_0} \) (the purple and red diagrams) increases and at \( t = 0.7 \), the \( \frac{1}{I_0} \) value will increase and will reach the approximately values 0.09 and 0.1, respectively. Then, with increasing \( t \), the \( \frac{1}{I_0} \) values (purple and red diagrams) reach to the maximum values of 2.95 and 2.9, respectively. Then, with increasing \( t \) at \( t = 2.4 \), the \( \frac{1}{I_0} \) values (red and purple diagrams) reach 0.4 and 0.45, respectively, and then increase with increasing \( t \) and at \( t = 3 \) the value of \( \frac{1}{I_0} \) reach to approximately values 0.7 and 0.8, respectively. It is understood that from Figures 16c and f when we take \( - \) or \( + \) sign in equation 15, for \( b = 500 \mu m, 0 \leq \frac{r}{R} \leq 3\text{and} \frac{2}{R} = t\left[1 \pm ve^{-t^2}\right] \) (blue and orange diagrams), at \( t = 0 \), the amount of intensity distribution of \( \frac{1}{I_0} \) is zero for both colors, which even with increasing \( t \), the value of \( \frac{1}{I_0} \) in the range of \( 0 \leq t \leq 2 \) is almost zero. Then, with increasing \( t \), it gradually increases and at \( t = 2.8 \), it reaches the maximum values of 2.98 and 2.99, respectively, and then with increasing \( t \), the value of \( \frac{1}{I_0} \) decreases and following at \( t = 3 \), the value of \( \frac{1}{I_0} \) reaches values of 1.35 and 1.30, respectively. Note that in drawing graphs of 16 and 17a, b and c we used: 

\[
\frac{r}{R} = t\left[1 - ve^{-t^2}\right],
\]

while in Figures 16 and 17, d and e and f we used: 

\[
\frac{r}{R} = t\left[1 + ve^{-t^2}\right].
\]

The result of our calculations has shown that, the yellow and blue diagrams have similar behavior with each other, similarly the green and red diagrams too. But with increasing \( b \) the value of \( \frac{1}{I_0} \) slightly increases. It is understood that from Figures 17a and d when we consider equation 25, for \( b = 300 \mu m \), we have orange (for \( + \) sign) and blue (for \( - + \) sign) diagrams that have similar behavior, that is, in the case of \( t = 0 \), the distribution of intensity value \( \frac{1}{I_0} \) is equal to unit.

Then, with increasing \( t \), the value of \( \frac{1}{I_0} \) in both of diagrams increases and at \( t = 1 \) reaches a maximum value of 1.00095. Then, with increasing \( t \), its value is reduced and at \( t = 3 \), both diagrams approach the unit value and similarly, in Figures 17a and d, (using equation 25) we have red (\( + \) sign) and violet (\( - \) sign) diagrams and from their comparing we find that the behavior is the opposite of the previous state, that is, at \( t = 0 \), the values of \( \frac{1}{I_0} \) of both diagrams are equal to one, and gradually decreases with \( t \) and at \( t = 1 \) reaches to the approximately value 0.99805 and then with increasing \( t \) the values of \( \frac{1}{I_0} \) for both diagrams are increased and at \( t = 3 \) they approach to the unit values. It is understood that from Figures 17b and e when we consider equation 25, for \( b = 400 \mu m \), we have orange (\( + \) sign) and blue (\( - + \) sign) diagrams that have the same behavior, that is, at \( t = 0 \), the distribution of intensity value \( \frac{1}{I_0} \) is zero. Then, with increasing \( t \), once at \( t = 0.65 \) value in the orange and blue diagrams increased and reach to 0.3 and 0.25, respectively. Then, both increases with increasing \( t \) and reach zero, but gradually increase at \( t = 1.9 \). And the orange and blue diagrams reach to maximum values of 2.9 and 2.91 at \( t=2.1 \), respectively, and then with increasing \( t \), the values of \( \frac{1}{I_0} \) for both orange and blue diagrams decreased and at \( t = 2.3 \) reach to 0.6 and 0.55, respectively. Then with increasing \( t \), the values of \( \frac{1}{I_0} \) for both diagrams are increased and at \( t = 2.7 \) the values of \( \frac{1}{I_0} \) are increased for these diagrams such that reach to 2.95 and 2.94, respectively. Then corresponding to the orange and blue diagrams the value of \( \frac{1}{I_0} \) decreases with increasing \( t \) and at \( t = 3 \), they reach to the values of approximately 1.3 and 1.35, respectively. It is understood that from Figures 17b and e, when we consider equation 25, for \( b = 400 \mu m \), we have the violet (\( + \) and red (\( - \) and \( + \) signs) diagrams that such, that at \( t = 0 \), the intensity distribution value of \( \frac{1}{I_0} \) for both diagrams are equal zero. Then, with increasing \( t \), at \( t=0.65 \), suddenly, the value
Figure 17: The distribution of intensity on the image plane for fields greater than the critical values in terms of the $t$ parameter and $\varepsilon = 0.5$ at the various values of $a$: $b = 300\mu m$, $b = 400\mu m$, $c: b = 500\mu m$ for $\frac{r}{R} = t(1 - ve^{-t})$ and $d: b = 300\mu m$, $e: b = 400\mu m$, and $f: b = 500\mu m$ for $\frac{r}{R} = t(1 + ve^{-t})$ related to two modes: 1: $I_{I_{0}} = \frac{1}{1 + \varepsilon} = \frac{1 + \varepsilon}{e^{t} - e^{t} + \varepsilon + (1 - 2e^{t})}$ (violet and orange).

of $\frac{I}{I_{0}}$ for both violet and red diagrams, reach to the values of 0.45 and 0.5, respectively, then both decrease with increasing $t$ and close to zero until they are increased from $t = 2.1$ to 3 and at $t = 3$ the value of $\frac{I}{I_{0}}$ for the violet and red diagrams are 0.85 and 0.8, respectively. It is understood that from Figures 17c and f, when we consider equation 25, for $b = 500\mu m$, we have orange ($+\cdot$) and blue ($-\cdot$) diagrams that have the same behavior, that is, at $t = 0$, the intensity distribution value of $\frac{I}{I_{0}}$ are equal to zero for both diagrams.

Then, with increasing $t$, at $t = 2.35$ the value of $\frac{I}{I_{0}}$ devoted
to the orange and blue diagrams increase and reach to the highest values 2.92 and 2.93, respectively, then both decrease with increasing t and at $t = 2.2$ the value of $\frac{I}{I_0}$ for orange and blue diagrams reach to the values of 0.6 and 0.57, respectively.

Then, with increasing t, the values of $\frac{I}{I_0}$ are increased for both orange and blue diagrams, and at $t = 2.75$, they reach to the 2.95 and 2.96, respectively. Then with increasing t, the value of $\frac{1}{I_0}$ is reduced in both diagrams and at $t = 3$ the values of $\frac{1}{I_0}$ devoted to blue and orange diagrams reach to the approximately values of 1.3 and 1.35, respectively. It is understood that from Figures 17c and f, when we consider equation 25, for $b = 500 \mu m$, we have the violet (+) and red (+) diagrams, such that, at $t = 0$, the intensity distribution values of $\frac{I}{I_0}$ are equal to zero for both diagrams. Then, with increasing $t$ to $t=2.1$, once the values of $\frac{1}{I_0}$ for both the violet and red diagrams are increased and reach to the values of 0.8 and 0.85, respectively. Note that in plotting the Figure 16 and 17a, b and c is used the relation: $\frac{I}{I_0} = t \left[ 1 - ve^{-t^2} \right]$ while in the Figures 16 and 17d, e and f is used the relation: $\frac{I}{I_0} = t \left[ 1 + ve^{+t^2} \right]$. The results of our calculations show that the choice of relations $\frac{I}{I_0} = t \left[ 1 - ve^{-t^2} \right]$ and $\frac{I}{I_0} = t \left[ 1 + ve^{+t^2} \right]$ have not much effects on the $\frac{1}{I_0}$ values. In addition, violet and red diagrams have similar behaviors, and in contrast, the orange and blue diagrams are similar, but with increasing b the value of $\frac{1}{I_0}$ slightly increases.

with these observations and the implications of such observation fields is discussed briefly and the results of calculating electromagnetic fields derived from proton radiography are:

i) The critical magnetic field for both focusing and defocusing modes for protons with energy of 14.7 MeV is more than protons with energy of 3 MeV, but with increasing $B$ for 14.7 and 3 MeV protons $B_0$ are reduced.

ii) The angle of deflection $\theta$ and the dimensionless dimension $\nu$ are increased with increasing $B_0$ and $b$.

iii) The position of the proton collision point with the image plane $(r)$ is a function of $B_0$ and $b$, which in some conditions increases with increasing $B_0$ and $b$, and in some other conditions decreases with increasing $B_0$ and $b$.

iv) The intensity distribution on the image plane for $\mu$ lower than the critical value and this quantity for fields more than the critical value with increasing $b$ is slightly increased.

REFERENCES


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